Spur Gears (Chapters 13 & 14)

Spur gears are used to transmit rotary motion between two parallel shafts. These gears are cylindrical, and the teeth are straight and parallel to the axis of rotation.

Fundamental Law of Toothed Gearing (see figure 10-3)

NN & TT represent normal and tangential at point of contact

$\overrightarrow{K_1 m_1}$ is a vector which represents the velocity of point K_1 on gear 1

$\overrightarrow{K_1 m_2}$ " of point K_1 on gear 2

from the figure we can write,

$$|\vec{m}_1| = |\omega_1| \cdot \overline{O_1 K_1} \qquad (1)$$

$$|\vec{m}_2| = |\omega_2| \cdot \overline{O_2 K_1} \qquad (2)$$

the ratio of angular velocities can be written as,

$$\frac{|\omega_2|}{|\omega_1|} = \frac{\overline{O_1 K_1}}{|\vec{m}_1|} \cdot \frac{|\vec{m}_2|}{\overline{O_2 K_1}} \qquad (3)$$

from similar triangles we have,

$$\frac{\overline{O_1K_1}}{|\vec{m}_1|} = \frac{\overline{O_1A}}{|\vec{N}_1|} \qquad (4)$$

$$\frac{|\vec{m}_2|}{\overline{O_2K_1}} = \frac{|\vec{N}_1|}{\overline{O_2B}} \qquad (5)$$

substituting eq. 4 & 5 into eq. 3,

$$\frac{|\omega_2|}{|\omega_1|} = \frac{\overline{O_1A}}{\overline{O_2B}} \qquad (6)$$

also from similar triangles,

$$\frac{\overline{O_1A}}{\overline{O_2B}} = \frac{\overline{O_1O}}{\overline{O_2O}} \qquad (7)$$

substituting eq 7 into eq. 6,

$$|\omega_1| \cdot \overline{O_1O} = |\omega_2| \cdot \overline{O_2O} \qquad (8)$$

Ideally we would like to treat gears as friction cylinders. (see figure 10-1). If there is no slip between cylinder we can write the following,

$$r_1\omega_1 = r_2\omega_2 \qquad (9)$$

Back to toothed gears. If the velocity ratio between gears is to be constant then the ω_1/ω_2 ratio must remain constant. Solving eq. 8 and 9 simultaneously and making use of the relationship

$$r_1 + r_2 = \overline{O_1O} + \overline{O_2O} \qquad (10)$$

we can write,

$$\overline{O_1O} = r_1 \qquad (11)$$

$$\overline{O_2O} = r_2 \qquad (12)$$

Therefore, point O is a fixed point through which pitch circles must be drawn.

* <u>Involutes</u> or <u>cycloids</u> fulfill the fundamental law!

* $|\vec{T}_2| - |\vec{T}_1|$ is the sliding velocity of one tooth relative to the other.

* When points K_1 and O coincide, pure rolling contact results.

Generation of Involute Tooth Profiles

Refer to figure 10-5

1. Pressure line is drawn in accordance with pressure angle

2. Base circle is constructed tangent to pressure line

3. Point E is located by making $\overset{\frown}{EB}$ equal to $\overline{OB}$

4. Step 3 is repeated for other points on the involute

* Think of involute profile as being generated by a point on a string as it is unwrapped.

* Cycloidal gear teeth not as popular because of the tooling required (see figure 10-6).

Gear Tooth Terminology

See figure 10-2 !

Pitch Equations

Circular Pitch - distance between corresponding points on adjacent teeth at pitch circle

$$p = \frac{\pi d}{N} \tag{14}$$

$p \triangleq$ circular pitch (in./tooth)
$d \triangleq$ pitch diameter (in.)
$N \triangleq$ number of teeth in gear

Diametral Pitch - number of teeth in gear per inch of pitch diameter

$$P_d = \frac{N}{d} \tag{15}$$

$P_d \triangleq$ diametral pitch (teeth/in.)

Useful relationship,

$$p \, P_d = \pi \tag{16}$$

Base Pitch - distance between corresponding points on adjacent teeth at base circle

$$p_b = p \cos\phi \qquad (17)$$

$p_b \triangleq$ base pitch (in./tooth)
$\phi \triangleq$ pressure angle (usually $14\frac{1}{2}°$ or $20°$)

Spur Gear Center Distance

$$c = r_1 + r_2 = \frac{N_1 + N_2}{2P_d} \qquad (18)$$

$c \triangleq$ center distance (in.)
$r \triangleq$ pitch radius (in.)

Basic Gear Systems (American Gear Manufactures Assouiation)

See figures 10-9 thru 10-12!

Bending Capacity of Spur Gear Teeth

See figure 10-18!

$$\sigma = \frac{Mc}{I}$$

$$I = \frac{bh^3}{12}$$

$$c = \frac{h}{2}$$

$$m = F_b\, l$$

$$\therefore \quad \sigma = \frac{F_b\, 6l}{bh^2} \qquad (19)$$

$h^2/6l$ is purely a geometric property of the size and shape of the tooth. Dividing this number by the pitch yeilds what is commonly referred to as the Lewis factor.

$$y = \frac{h^2}{6lp} \qquad (20)$$

$y \hat{=}$ Lewis factor of form factor

Substituting eq. 20 into 19,

$$F_b = \sigma b y p \qquad (21)$$

This equation gives the tangential load the tooth can carry. Sometimes the form factor is expressed as,

$$Y = \pi y$$

and since,

$$p = \frac{\pi}{P_d}$$

eq. 21 can be written as,

$$F_b = \sigma b \frac{Y}{P_d} \qquad (22)$$

For design tables 10-1 and 10-2 can be used to select materials, pitch, pressure angle and face width.

Limit Load for Wear

* Contact stresses between cylinders

$$\sigma_{max} = 0.591 \left[\left(\frac{P' E_1 E_2}{E_1 + E_2} \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right] \quad (23)$$

$P' \triangleq$ load (lb/in)
$R \triangleq$ cylinder radius (in)
$E \triangleq$ Young's modulus (lb/in²)
$\sigma_{max} \triangleq$ maximum contact stress (lb/in²)

When contact occurs at the pitch diameter of either gear,

$$R_1 = \frac{d_1}{2} \sin \phi \quad (24)$$

$$R_2 = \frac{d_2}{2} \sin \phi$$

Using the relationship $\frac{d_1}{N_1} = \frac{d_2}{N_2}$, eq. 24 can be rewritten as,

$$R_2 = \frac{N_2 d_1}{2 N_1} \sin \phi \quad (25)$$

substituting equations 24 and 25 into equation 23,

$$\sigma^2 = 0.35\left(\frac{PE_1E_2}{E_1+E_2}\right)\left(\frac{2}{d_1 \sin\phi}\right)\left(1+\frac{N_1}{N_2}\right) \quad (26)$$

the wear load is defined as follows,

$$F_w = P'b \quad (27)$$

$F_w \triangleq$ wear load (lb·in)
$b \triangleq$ face width (in)

solving for eq. 26 for F_w,

$$F_w = \frac{\sigma^2 b d_1 \sin\phi}{1.40}\left(\frac{1}{E_1}+\frac{1}{E_2}\right)\left(\frac{2N_2}{N_1+N_2}\right) \quad (28)$$

this equation is further simplified by introducing two new variables Q and K,

$$Q = \frac{2N_2}{N_1+N_2} \quad (29)$$

$$K = \left(\frac{\sigma^2 \sin\phi}{1.4}\right)\left(\frac{1}{E_1}+\frac{1}{E_2}\right) \quad (30)$$

where σ represents the surface endurance limit in compression.

The final design equation is as follows,

$$F_w = d_1 b Q K \tag{31}$$

Table 10-2a provides values for allowable work stress and for K.

Additional Considerations in Spur Gears

Errors in Forming
Spring Constants
Effective Mass
Dynamic Loading
Mass Reduction
Lubrication
Mounting
Backlash
Undercutting
Internal or Annular Gears

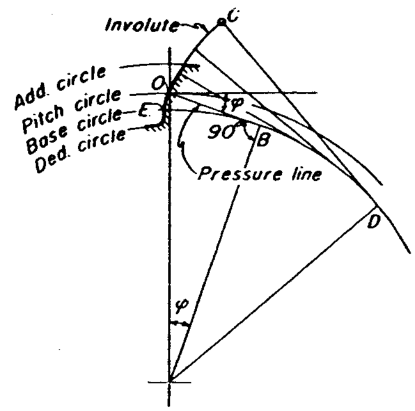

Figure 10-5 Drawing board layout for the involute.

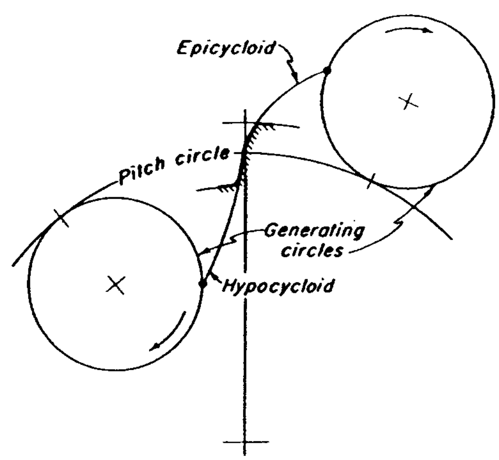

Figure 10-6 Generation of epicycloid and hypocycloid.

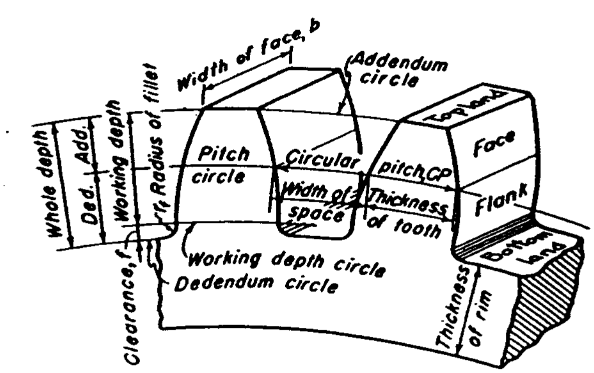

Figure 10-2 Principal parts of gear teeth.

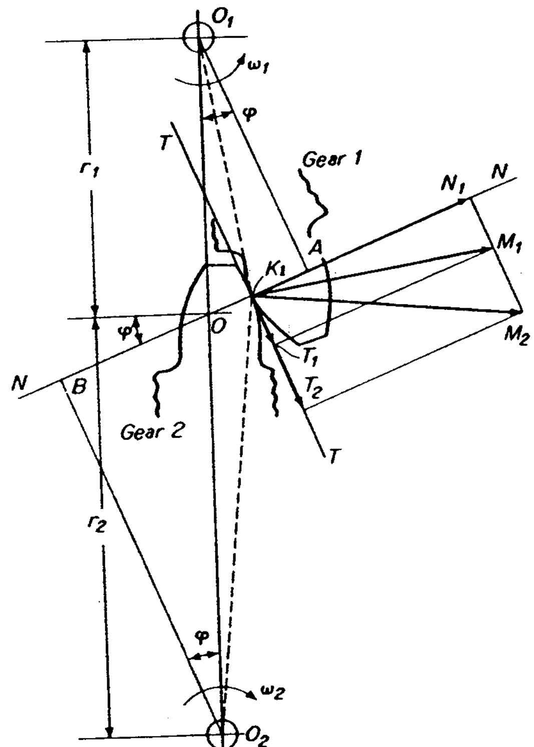

Figure 10-3 Component of velocity normal to tooth surface at point of contact is same for both gears.

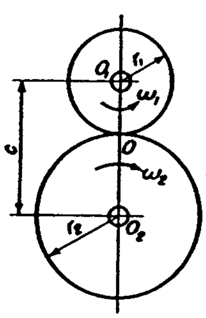

Figure 10-1 Friction cylinders.

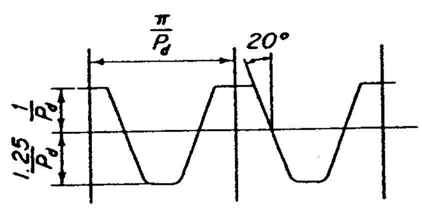

Figure 10-9 Basic rack for 20° full-depth involute system. 25° system is similar except for change in angle.

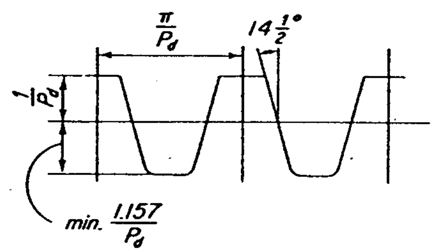

Figure 10-10 Basic rack for $14\frac{1}{2}°$ full-depth involute system.

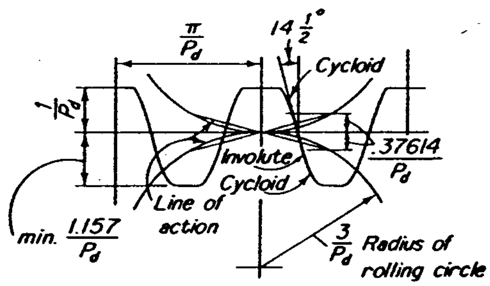

Figure 10-11 Basic rack for $14\frac{1}{2}°$ composite system.

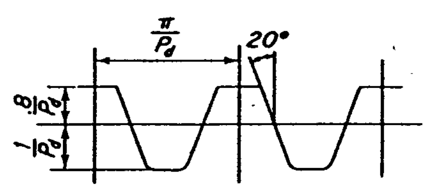

Figure 10-12 Basic rack for 20° stub-tooth involute system.

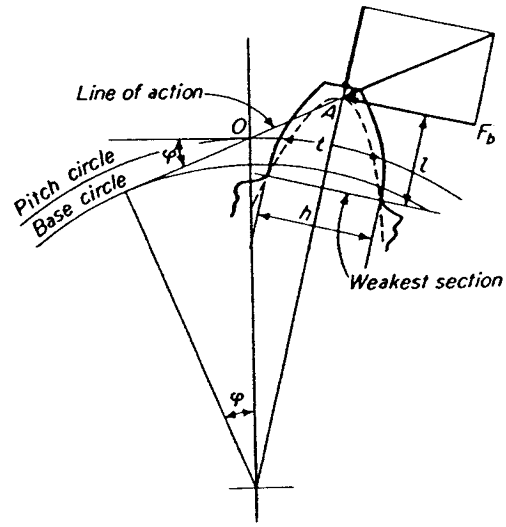

Figure 10-18 Beam strength of gear tooth.

Table 10-1 FORM OR LEWIS FACTOR y FOR SPUR GEARS WITH LOAD AT TIP OF TOOTH

No. of Teeth	*14-1/2° Full Depth*	*20° Full Depth*	*20° Stub*	*No. of Teeth*	*14-1/2° Full Depth*	*20° Full Depth*	*20° Stub*	*No. of Teeth*	*14-1/2° Full Depth*	*20° Full Depth*	*20° Stub*
10	0.056	0.064	0.083	19	0.088	0.100	0.123	43	0.108	0.126	0.147
11	0.061	0.072	0.092	20	0.090	0.102	0.125	50	0.110	0.130	0.151
12	0.067	0.078	0.099	21	0.092	0.104	0.127	60	0.113	0.134	0.154
13	0.071	0.083	0.103	23	0.094	0.106	0.130	75	0.115	0.138	0.158
14	0.075	0.088	0.108	25	0.097	0.108	0.133	100	0.117	0.142	0.161
15	0.078	0.092	0.111	27	0.099	0.111	0.136	150	0.119	0.146	0.165
16	0.081	0.094	0.115	30	0.101	0.114	0.139	300	0.122	0.150	0.170
17	0.084	0.096	0.117	34	0.104	0.118	0.142				
18	0.086	0.098	0.120	38	0.106	0.122	0.145	rack	0.124	0.154	0.175

Table 10-2 ALLOWABLE WORKING STRESSES, PSI, AND VALUES OF K FOR GEAR MATERIALS

Material	*Hardness*		*Bending, σ, psi*		*Compression, σ_{ec}, psi*	*K, $14\frac{1}{2}°$, psi*	*K, 20°, psi*
	BHN	*R_c*	*Spur, Helical*	*Bevel*			
Cast iron	160–200		5,000	3,000	50,000	56	76
Cast iron	210–245		7,000	4,000	60,000	80	110
Steel	160–200		20,000	10,000	60,000	43	59
Steel	210–245		22,000	11,000	70,000	58	80
Steel	302–351	33–38	32,000	15,000	100,000	119	163
Flame or induction hardened		48–53	35,000	15,000	160,000	305	417
Carburized or case hardened		58–63	55,000	30,000	200,000	477	651

Values for K are calculated for both gears of the same material. For combinations of steel and cast iron or bronze, see Problem 12.

Table 10-2A ALLOWABLE WORKING STRESSES, AND VALUES OF K FOR GEAR MATERIALS, SI UNITS

Material	*Hardness*		*Bending, σ, MPa*		*Compression, σ_{ec}, MPa*	*K, $14\frac{1}{2}°$, MPa*	*K, 20°, MPa*
	BHN	*R_c*	*Spur, Helical*	*Bevel*			
Cast iron	160–200		34.5	20.7	344.8	0.386	0.524
Cast iron	210–245		48.3	27.6	413.8	0.552	0.759
Steel	160–200		137.9	69.0	413.8	0.297	0.407
Steel	210–245		151.7	75.9	482.8	0.400	0.552
Steel	302–351	33–38	220.7	103.4	689.7	0.821	1.124
Flame or induction hardened		48–53	241.4	103.4	1,103.4	2.103	2.876
Carburized or case hardened		58–63	379.3	206.9	1,379.3	3.290	4.490

Values for K apply when both gears are of same material.

For combinations of steel and cast iron or bronze, see Problem 12.

Stress values are those of Table 10-2 divided by 145, being the conversion factor from psi to MPa.