Shaft Design

I. Design Criterion

* Shaft Rigidity Guidelines

Case	Lateral Deflection	Torsional Deflection
transmission shaft	0.01 L	- - - - -
line shaft	0.01 L	- - -
machinery	0.002 L	0.1°L or 0.5° L/d
machine tools	- - - -	1°

* Loading Checklist

Type
- bending forces & moments
- torque } Thin hollow shafts may buckle.
- axial

Potential Sources
- intended operating loads
- possible operating loads
- starting and stopping torques
- shaft weight
- resonance (lateral and torsion)

key fits
press fits
unbalance
misalignment
seals
restricted thermal growth
centrifugal
fluid or windage losses
developed pressures
fluctuating forces (most often due to gear tooth mesh)
impacts

* Materials

Most Common Material
AISI 1020-1050 or 1100

Higher Strength Alloys
AISI 1340-50, 3140-50, 4140, 4340, 5140, 8650

Surface Hardening
induction heating of medium carbon steels
carburizing of low carbon steels

General Practice

0 - 3.5 in shafts ⇒ cold rolled & drawn steels or cold drawn and ground

3.5in - 6 in shafts ⇒ hot rolled & machined

6in > shafts ⇒ forged & machined

* Lowering Stress Concentrations

See figure 37-1 !

Required Shaft Diameters

Torsional Rigidity

$$\theta = \frac{TL}{JG} \qquad (1)$$

Torsional Strength

$$\tau = \frac{Tc}{J} \qquad (2)$$

Bending Strength

$$\sigma = \frac{Mc}{I} \qquad (3)$$

Using failure theories and fatigue methods as developed previously in class the equations in table 37-3 can be derived.

Looking at the development of combined torsion and bending diameter equation,

$$\tau = \frac{Tc}{J} \quad (4)$$

$$\sigma = \frac{Mc}{I} \quad (5)$$

$$J = \frac{\pi d^4}{32} \quad (6)$$

$$I = \frac{\pi d^4}{64} \quad (7)$$

$$c = \frac{d}{2} \quad \text{(for maximum stress level)} \quad (8)$$

DET (von Mises)

$$\left[(\sigma_1 - \sigma_2)^2 + \sigma_1^2 + \sigma_2^2\right] \geq 2\sigma_f^2 \quad (9)$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}} \quad (10)$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}} \quad (11)$$

Rewriting eq. 10 & 11 in terms of eq. 4 & 5,

$$\sigma_1 = \frac{\sigma}{2} + \left[\left(\frac{\sigma}{2}\right)^2 + \tau^2 \right]^{\frac{1}{2}} \qquad (12)$$

$$\sigma_2 = \frac{\sigma}{2} - \left[\left(\frac{\sigma}{2}\right)^2 + \tau^2 \right]^{\frac{1}{2}} \qquad (13)$$

Substituting eqs. 12 & 13 into eq. 9,

$$4\left(\frac{\sigma^2}{4} + \tau^2\right) + \left(\frac{\sigma}{2}\right)^2 + 2\left[\left(\frac{\sigma}{2}\right)^2 + \tau^2\right]^{\frac{1}{2}} + \left(\frac{\sigma}{2}\right)^2 + \tau^2 + \left(\frac{\sigma}{2}\right)^2 - 2\left[\left(\frac{\sigma}{2}\right)^2 + \tau^2\right]^{\frac{1}{2}}$$

$$+ \left(\frac{\sigma}{2}\right)^2 + \tau^2 \Big] \geq 2\sigma_f^2. \qquad (14)$$

Simplifying,

$$\left[\sigma^2 + 3\tau^2\right]^{\frac{1}{2}} \geq \sigma_f \qquad (15)$$

and since eqs. 4 & 5 can be written as,

$$\tau = \frac{16T}{\pi d^3} \qquad (16)$$

$$\sigma = \frac{32M}{\pi d} \qquad (17)$$

and if the failure stress is expressed as,

$$\sigma_f = \frac{\sigma_{yp}}{FS} \qquad (18)$$

we can rearrange eq. 15 and solve for d,

$$d = \left[\frac{32\, FS}{\pi\, \sigma_{yp}} \left(m^2 + \frac{3T^2}{4} \right)^{\frac{1}{2}} \right]^{\frac{1}{3}} \qquad 19$$

Similar derivations can be extended to the other equations in table 37-3.

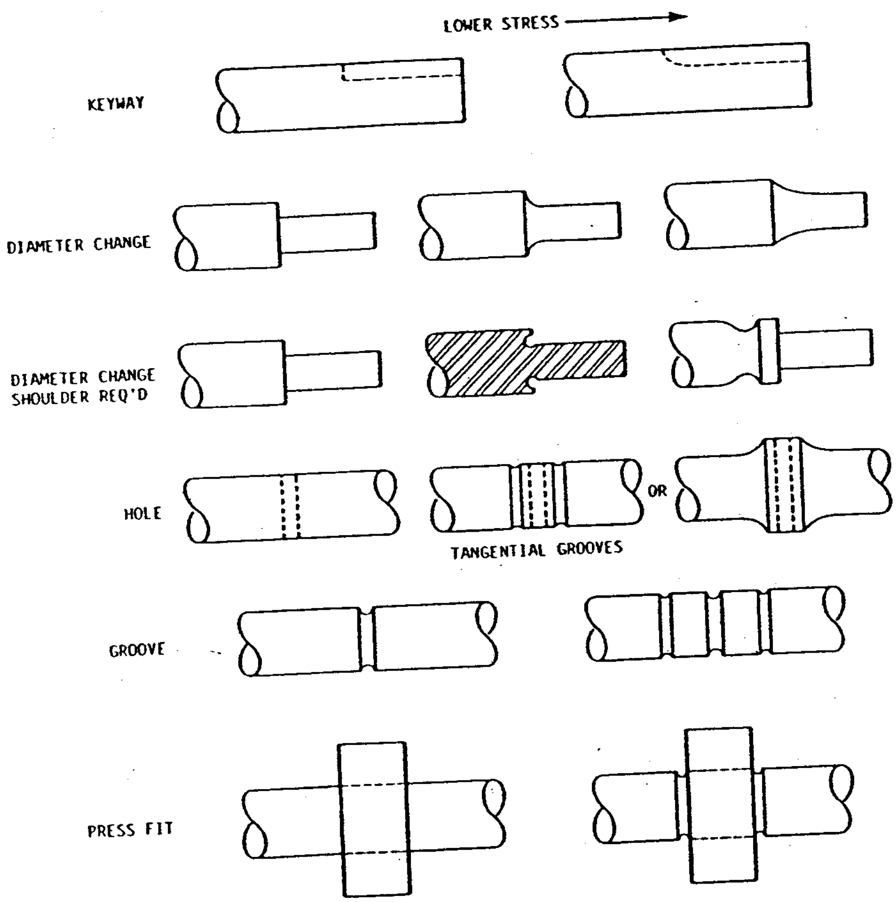

FIG. 37-1 Lowering stress concentrations.

TABLE 37-3 Required Shaft Diameter for Various Loadings†

Loading	Diameter	Equation	Remarks
	Static		
Torsion	$d = \left(\dfrac{16\sqrt{3}nT}{\pi S_y}\right)^{1/3}$	(37-11)	
Bending	$d = \left(\dfrac{32nM}{\pi S_y}\right)^{1/3}$	(37-14)	
Torsion and bending	$d = \left[\dfrac{32n}{\pi S_y}\left(M^2 + \dfrac{3T^2}{4}\right)^{1/2}\right]^{1/3}$	(37-16)	
	Dynamic		
Reversed bending and steady torque	$d = \left[\dfrac{32n}{\pi}\left(\dfrac{M}{S_e} + \dfrac{\sqrt{3}}{2}\dfrac{T}{S_u}\right)\right]^{1/3}$	(37-18)	Ref. [37-13]
	$d = \left\{\dfrac{32n}{\pi}\left[\left(\dfrac{M}{S_e}\right)^2 + \dfrac{3}{4}\left(\dfrac{T}{S_y}\right)^2\right]^{1/2}\right\}^{1/3}$	(37-19)	Distortion energy—elliptic; new ASME standard per Refs. [37-3] and [37-18]
	$d = \left\{\dfrac{32n}{\pi}\left[\left(\dfrac{M}{S_e}\right)^2 + \left(\dfrac{T}{S_y}\right)^2\right]^{1/2}\right\}^{1/3}$	(37-20)	Maximum shear—Soderberg; Ref. [37-14] and [37-15]
Mean plus alternating: bending and torque	$d = \left(\dfrac{32n}{\pi}\left\{\left[\left(\dfrac{M_a}{S_e}\right)^2 + \dfrac{3}{4}\left(\dfrac{T_a}{S_e}\right)^2\right]^{1/2} + \left[\left(\dfrac{M_m}{S_u}\right)^2 + \dfrac{3}{4}\left(\dfrac{T_m}{S_u}\right)^2\right]^{1/2}\right\}\right)^{1/3}$	(37-21)	Ref. [37-16]
	$d = \left\{\dfrac{32n}{\pi}\left[\left(\dfrac{M_a}{S_e} + \dfrac{M_m}{S_y}\right)^2 + \left(\dfrac{T_a}{S_e} + \dfrac{T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3}$	(37-22)	Maximum shear—Soderberg; Ref. [37-15]
Mean plus alternating: bending, torque, and axial loads	$d = \left\{\dfrac{32n}{\pi S_e}\left[\left(M_a + \dfrac{P_a d}{2}\right)^2 + \dfrac{3T_a^2}{4}\right]^{1/2} + \dfrac{32n}{\pi S_u}\left[\left(M_m + \dfrac{P_m d}{2}\right)^2 + \dfrac{3T_m^2}{4}\right]^{1/2}\right\}^{1/3}$	(37-23)	Not explicit in d; use iterative methods to solve; Refs. [37-16], [37-17]

†For solid round ductile material; unless noted otherwise, the equations are based on distortion energy, Goodman.