Lecture 9

Variable Loading (7-1 thru 7-5)

* Fluctuating stress levels, as with the surface of rotating shafts, result in cummulative damage to the part. These parts fail at stress below the ultimate strength, and even below the yeild strength.

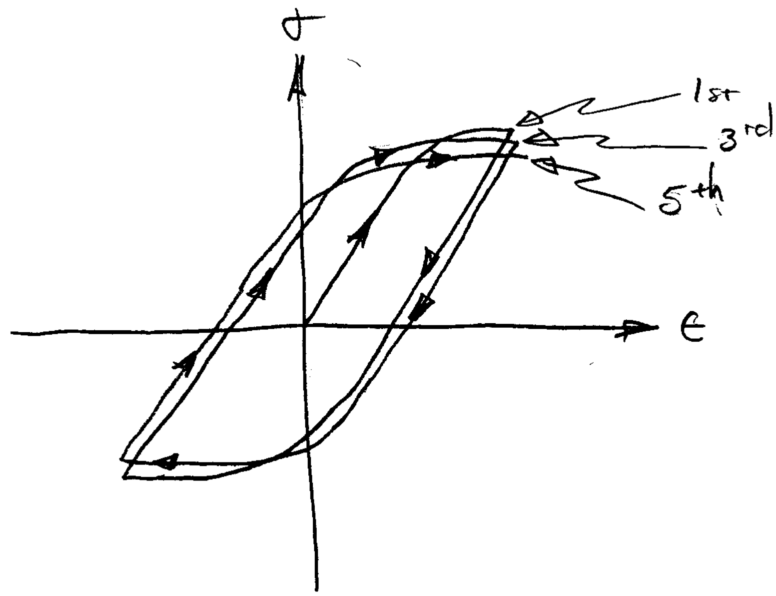
* Fatigue failure begins with a small crack that develops at a point of discontinuity in the material. The crack propegates resulting in failure of the material / part.

Strain-Life Theory of Fatigue Failure

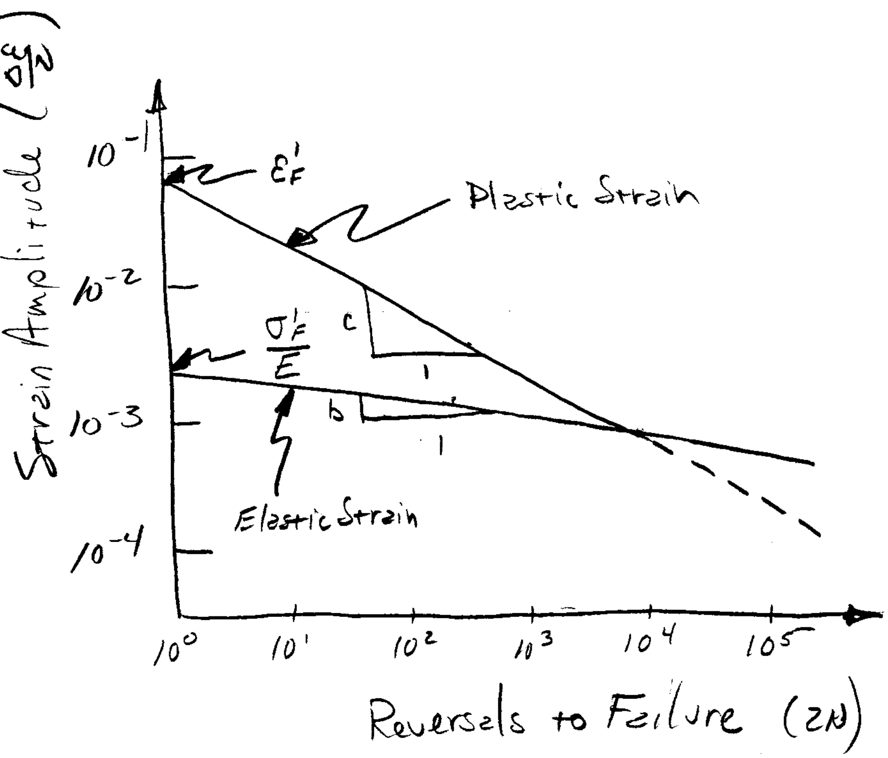
* Theory is useful for explaining nature of fatigue.

* Stress at the tip of the propagating crack exceeds the elastic limit and plastic deformation occurs.

* Strength decreases with stress repetition as applied through completely reversed bending.



True Stress-Strain Hysteresis Loops.



Log-Log Plot of Fatigue Life

$\varepsilon'_F \triangleq$ fatigue ductility coefficient
(Corresponds to fracture in one reversal.)

$\sigma'_F \triangleq$ fatigue strength coefficient
(Corresponds to fracture in one reversal.)

$c \triangleq$ fatigue ductility exponent

$b \triangleq$ fatigue strength exponent

* Total same is sum of elastic and plastic strains.

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{\Delta \varepsilon_p} + \frac{\Delta \varepsilon_p}{2} \tag{1}$$

The plasic-strain line equation is,

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_F (2N)^c \tag{2}$$

and the equation of the elastic strain line is,

$$\frac{\Delta \varepsilon_e}{2} = \frac{\sigma'_F}{E} (2N)^b \tag{3}$$

Equation 1 can be rewritten as

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma_F'}{2} (2N)^b + \varepsilon'_F (2N)^c \tag{4}$$

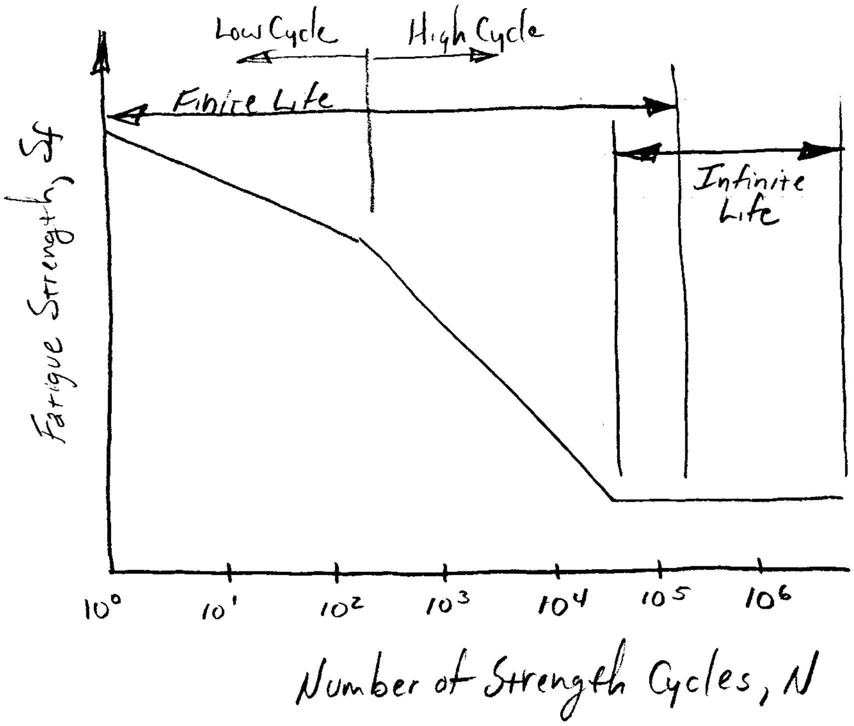
* See Table 7-1 in text for constants of various materials.

* Equation 4 is of little use to the designer as it difficult to determine the total strain at the tip of a propagating crack.

Stress Life

S-N Diagrams

R.R. Moore rotating beam machine for completely reversed bending.



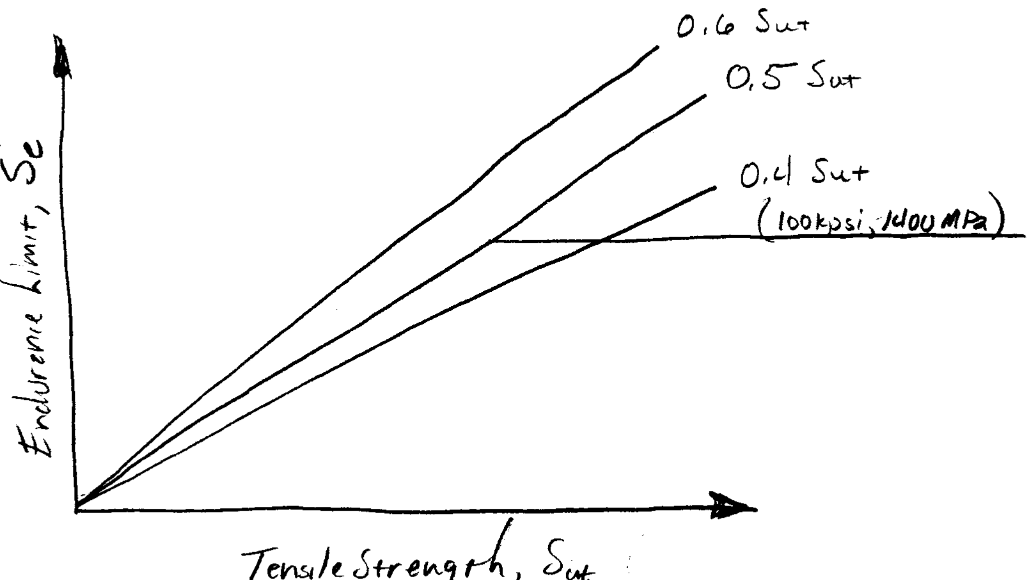
* 1-1000 cycles is considered low cycle fatigue.
* >1000 cycles is considered high cycle fatigue.

* Division between finite and infinite life is 10^6 to 10^7 cycles.

Endurence Limit

* For steels the endurence limit can be approximated as,

$$S_e' = \begin{cases} 0.504\, S_{ut} & S_{ut} \le 200\,\text{kpsi or } 1400\,\text{MPa} \\ 100\,\text{kpsi} & S_{ut} > 200\,\text{kpsi} \\ 1400\,\text{MPa} & S_{ut} > 1400\,\text{MPa} \end{cases}$$



Fatigue Strength

Low cycle fatigue

Assume the equation for the S-N line takes the form,

$$S_f = a N^b \tag{5}$$

at 10^3 cycles,

$$(S_f)_{10^3} = a(10^3)^b = a(10)^{3b} = f\,S_{ut} \tag{6}$$

or,

$$f = \frac{a}{S_{ut}}(10)^{3b} \tag{7}$$

High cycle fatigue

In high-cycle fatigue the stress level is perdominantly elastic,

$$\sigma_a = \frac{\Delta \epsilon_t E}{2} \tag{8}$$

Equation 3 becomes,

$$\sigma_a = \sigma_f'(2N)^b \tag{9}$$

and solving for b,

$$b = \frac{1}{\log 2N}\log\frac{\sigma_f'}{\sigma_a} = -\frac{\log\left(\frac{\sigma_f'}{S_e}\right)}{\log(2N_e)} \tag{10}$$

Multiplying both sides by 3 and substituting $N_e = 10^6$ cycles,

$$3b = \log\left(\frac{\sigma_f'}{S_e}\right)^{-\frac{1}{1.2}} \qquad (11)$$

f from eq. 7 becomes,

$$f = \frac{a}{S_{ut}}(10)^{3b} = \frac{2^b \sigma_f'}{S_{ut}}\left(\frac{\sigma_f'}{S_e}\right) \qquad (12)$$

The fatigue coefficient σ_f' can be approximated by

$$\sigma_f' = S_{ut} + 58.8 \text{ kpsi} \qquad (13)$$

Equation 12 can be solved when S_{ut} and S_e are given for a material since b can be calculated from eq. 11.

* For most materials S_f (failure strength at 1000 cycles) can be estimated as

$$S_f = 0.9\, S_{ut} \qquad (14)$$

as the above analysis shows. A rough estimate for S_e (failure strength at 10^6 cycles) is

$$S_e = 0.5\, S_{ut} \qquad (15)$$

* the latter is a rough approximation for S_e and should only be used when the endurance limit of the material is not known.

Another way to find S_f for a finite design life is to write eq. 5 as

$$\log S_f = \log a + b \log N \qquad (16)$$

If this line is to intersect S_e at 10^6 cycles and $0.9\, S_{ut}$ at 10^3 cycles then a and b are defined as,

$$a = \frac{(0.9\, S_{ut})^2}{S_e} \qquad (17)$$

$$b = -\frac{1}{3} \log \frac{0.9\, S_{ut}}{S_e} \qquad (18)$$

* Note that constant a depends on the units used.

For completely reversed bending of stress amplitude σ_a the number of cycles to failure is give as

$$N = \left(\frac{\sigma_a}{a} \right)^{\frac{1}{b}} \qquad (19)$$

Example

Estimate the endurance limit, fatigue strength at 10^4 cycles and the estimated life for completely reversed stress of 55 kpsi for AISI 1040 steel.

From Table A-21 $S_y = 54$ kpsi and $S_{ut} = 86$ kpsi for 1040 annealed steel.

$$S_e' = 0.5\, S_{ut} = 0.5(86\text{ kpsi}) = 43\text{ kpsi}$$

$$a = \frac{(0.9\, S_{ut})^2}{S_e} = \frac{[(0.9)(86\text{ kpsi})]^2}{43\text{ kpsi}} = 139\text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.9\,S_{ut}}{S_e} = -\frac{1}{3}\log\left[\frac{(0.9)(86\text{kpsi})}{43\text{kpsi}}\right] = -0.0851$$

$$S_f = (139\text{ kpsi})(10^4)^{-0.0851} = 63.5\text{ kpsi}$$

$$N = \left(\frac{\sigma_a}{a}\right)^{\frac{1}{b}} = \left(\frac{55\text{kpsi}}{139\text{ kpsi}}\right)^{\frac{-1}{0.0851}} = 56{,}400\text{ cycles}$$