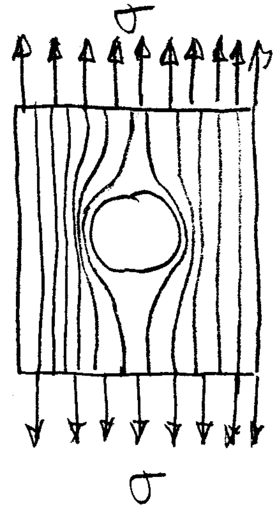
Lecture 6

Stress Concentrations (4-14, 4-15, 4-17 & 4-20)



* Visualize stress flow patterns -- narrow spacings of stress trajectories indicate regions with stress concentrations.

Theoretical Stress - Concentration Factor

$$K_t = \frac{\sigma_{max}}{\sigma_o} \quad \text{(normal stress)} \qquad (1)$$

$$K_{ts} = \frac{\tau_{max}}{\tau_o} \qquad (2)$$

Stress concentration factors (K_t or K_{ts}) relate the actual maximum stress at a stress discontinuity to the nominal stress. The subscript t means that the stress concentration factor depends on the geometry only.

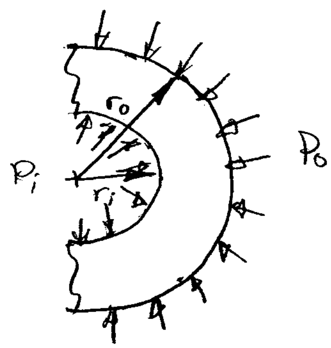
* See stress concentration factors in Appendix A, Fig A-15-1 through A-15-15.

Stresses in Cylinders

* Stress distribution in cylinders can be estimated using either a "thick-walled" or "thin-walled" approach.

* "Thin-walled" relationships should be used when the wall thickness is less then or equal to 1/20 of the wall radius.

Thick-Walled



Tangential Stress

$$\sigma_t = \frac{P_i r_i^2 - P_o r_o^2 - r_i^2 r_o^2 (P_o - P_i)/r^2}{r_o^2 - r_i^2} \quad (3)$$

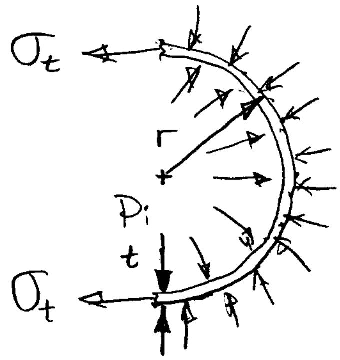
Radial Stress

$$\sigma_r = \frac{P_i r_i^2 - P_o r_o^2 + r_i^2 r_o^2 (P_o - P_i)/r^2}{r_o^2 - r_i^2} \quad (4)$$

Longitudinal Stress

$$\sigma_z = \frac{(P_i - P_o) r_i^2}{r_o^2 - r_i^2} \quad (5)$$

Thin-Walled



Tangential Stress

$$\sigma_{t,avg} = \frac{P_i - P_o\, d_i}{2t} \tag{6}$$

$$\sigma_{t,max} = \frac{(P_i - P_o)(d_i + t)}{2t} \tag{7}$$

Longitudinal Stress

$$\sigma_l = \frac{(P_i - P_o)\, d_i}{4t} \tag{8}$$

Example

A pressure vessel made of tubing has an outside diameter of 10.0 in. The wall thickness is 0.375 in. If the vessel is subjected to an internal pressure of 5000 psi. What is the difference in tangential stress calculated und the "thick-walled" versus "thin-walled" approach?

Thick-Walled

$$r_i = 5.000\,in - 0.375\,in = 4.625\,in$$

$$r_o = 5.000\,in$$

$$\sigma_t = \frac{P_i\left(r_i^2 + r_o^2\right)}{r_o^2 - r_i^2}$$

$$\sigma_t = \frac{\left(5000\,\frac{lb}{in^2}\right)\left((5.000\,in)^2 + (4.625\,in)^2\right)}{(5.000\,in)^2 - (4.625\,in)^2}$$

$$\sigma_t = 64,300\,\frac{lb}{in^2}$$

Thin-Walled

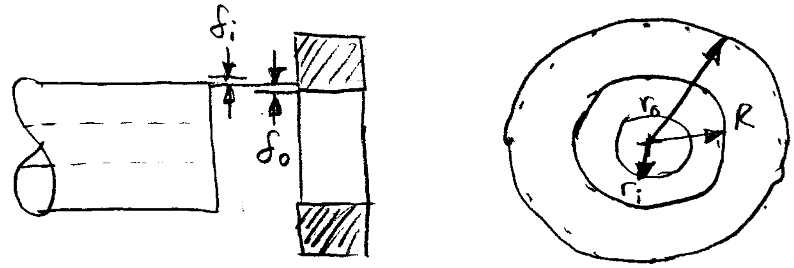
$$\sigma_{t,max} = \frac{P_i\left(d_i + t\right)}{2t}$$

$$\sigma_{t,max} = \frac{\left(5000\,\frac{lb}{in^2}\right)\left(9.250\,in + 0.375\,in\right)}{2(0.375\,in)}$$

$$\sigma_{t,max} = 64,200\,\frac{lb}{in^2}$$

Difference $\Rightarrow$ 0.156 %

Press and Shrink Fits



* When we design for interference in a fit, there exists a contact pressure p causing a radial stress at the contacting surface of either member.

* From the "thick walled" pressure vessel analysis the tangential stress in inner member is

$$\sigma_{it} = -p \, \frac{R^2 + r_i^2}{R^2 - r_i^2} \qquad (9)$$

and for the outer member

$$\sigma_{ot} = p \, \frac{r_o^2 + R^2}{r_o^2 - R^2} \qquad (10)$$

at a radius R of the contacting surfaces.

* The tangential strain at R for the outer cylinder is the change in circumference,

$$\epsilon_{ot} = \frac{2\pi(R+\delta_o) - 2\pi R}{2\pi R} \qquad (11)$$

Or,

$$\epsilon_{ot} = \frac{\delta_o}{R} \qquad (12)$$

We also know that

$$\epsilon_{ot} = \frac{\sigma_{ot}}{E_o} - \frac{\nu_o \sigma_{or}}{E_o} \qquad (13)$$

By substituting the tangential stress and radial stress relationships for "thick walled" vessels for the outside ring where,

$$\sigma_t = \frac{pR^2 + p r_o^2}{r_o^2 - R^2} = p\left(\frac{r_o^2 + R^2}{r_o^2 - R^2}\right) \qquad (14)$$

$$\sigma_r = \frac{pR^2 - p r_o^2}{r_o^2 - R^2} = -p \qquad (15)$$

Substituting into the equation for ϵ_{ot},

$$\epsilon_{ot} = \frac{P}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2}\right) + \frac{P}{E_o}\nu_o \qquad (16)$$

or

$$\epsilon_{ot} = \frac{P}{E_o}\left[\left(\frac{r_o^2 + R^2}{r_o^2 - R^2}\right) + \nu_o\right] \qquad (17)$$

Setting eq. 17 equal to 12,

$$\frac{\delta_o}{R} = \frac{P}{E_o}\left[\left(\frac{r_o^2 + R^2}{r_o^2 - R^2}\right) + \nu_o\right] \qquad (18)$$

or

$$\delta_o = \frac{pR}{E_o}\left[\left(\frac{r_o^2+R^2}{r_o^2-R^2}\right)+\nu_o\right] \qquad (19)$$

Using a similar approach for the inner part,

$$\delta_i = -\frac{pR}{E_i}\left[\left(\frac{R^2+r_i^2}{R^2-r_i^2}\right)-\nu_i\right] \qquad (20)$$

The total interference in the fit is is the sum of δ_i and δ_o, or,

$$\delta_t = |\delta_i| + |\delta_o| \qquad (21)$$

or

$$\delta_t = \frac{pR}{E_o}\left(\frac{r_o^2+R^2}{r_o^2-R^2}+\nu_o\right) + \frac{pR}{E_i}\left(\frac{R^2+r_i^2}{R^2-r_i^2}-\nu_i\right) \qquad (22)$$

If the mating parts are of the same material then eq. 22 simplifies to

$$\delta_t = \frac{pR}{E}\left(\frac{r_o^2+R^2}{r_o^2-R^2}+\frac{R^2+r_i^2}{R^2-r_i^2}\right) \qquad (23)$$

which can be solved for p. as

$$p = \frac{E\delta_t}{R}\left[\frac{(r_o^2-R^2)(R^2-r_i^2)}{2R^2(r_o^2-r_i^2)}\right] \qquad (24)$$

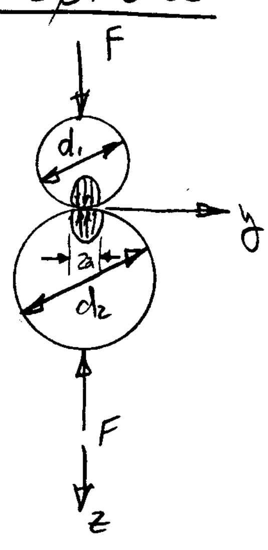
Contact Stresses

* Hertzian Stresses – contact stress between two bodies of double radius curvature, shperes and rotating cylinders.

* What types of mechanisms see contact stresses?

1. roller bearings
2. gear teeth
3. cams
4. tappets

Two Spheres



Radius of Contact

$$a = \left[\left(\frac{3F}{8}\right)\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{\frac{1}{d_1}+\frac{1}{d_2}}\right]^{\frac{1}{3}} \qquad (25)$$

Maximum Contact Pressure

$$p_{max} = \frac{3F}{2\pi a^2} \qquad (26)$$

Maximum Principle Stresses

$$\sigma_x = \sigma_y = -p_{max}\left[\left(1-\frac{z}{a}\tan^{-1}\frac{1}{\frac{z}{a}}\right)(1+\mu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right] \qquad (2$$

$$\sigma_z = \frac{p_{max}}{1+\frac{z^2}{a^2}} \qquad (28)$$

z in this case is the distance from the point of contact along the z-axis.

Shear Stress

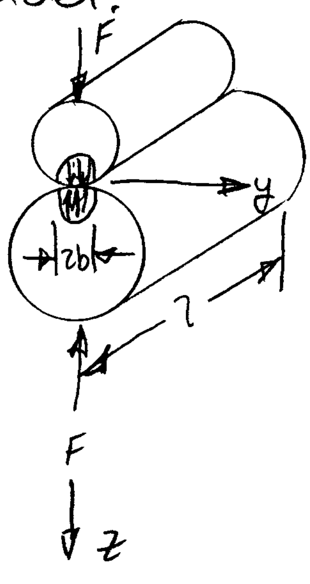
$$\tau_{xz} = \tau_{yz} = \frac{\sigma_x - \sigma_z}{2} = \frac{\sigma_y - \sigma_z}{2} \tag{29}$$

$$\tau_{xy} = 0 \quad (\text{since } \sigma_x = \sigma_y) \tag{30}$$

See Fig. 2-33 in text!

Two Cylinders

Cylinders are treated similarly with the exception that there is line contact instead of point contact. Two new variables are introduced, l and b, that describe the geometry of the contact.



$$b = \left[\left(\frac{2F}{\pi l}\right) \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{\frac{1}{d_1} + \frac{1}{d_2}} \right]^{\frac{1}{2}} \tag{31}$$

$$p_{max} = \frac{2F}{\pi b l} \tag{32}$$

$$\sigma_x = -2\nu p_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \frac{z}{b}\right) \qquad (33)$$

$$\sigma_y = -p_{max}\left[\left(2-\frac{1}{1+\frac{z^2}{b^2}}\right)\left(1+\frac{z^2}{b^2}\right)^{\frac{1}{2}} - 2\frac{z}{b}\right] \qquad (34)$$

$$\sigma_z = \frac{-p_{max}}{\sqrt{1+\frac{z^2}{b^2}}} \qquad (35)$$

See Figs. 2-35 and 2-36 from text!