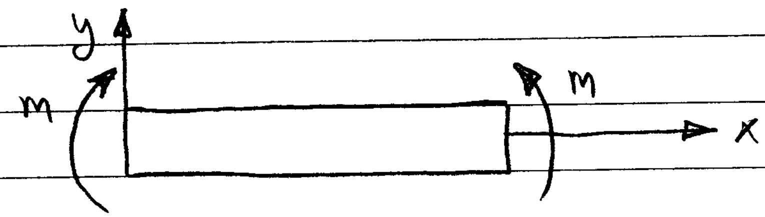
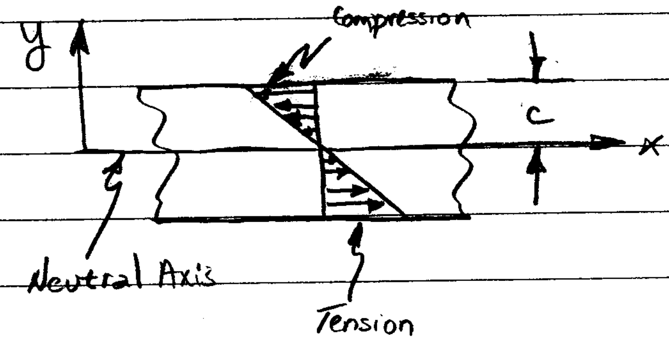
# Lecture 5

## Normal Stress in Flexure (4-10 to 4-13)




### Bending Stress

$$\sigma = \frac{Mc}{I} \qquad (1)$$

where

$\sigma \triangleq$ bending stress

$M \triangleq$ bending moment

$c \triangleq$ distance from neutral axis to extreme fiber

$I \triangleq$ second moment of area about z-axis

### Second Moment of Cross-Section

$$I = \int y^2 \, dA \qquad (2)$$

where

$A \triangleq$ cross-sectional area

## Section Modulus

$$\sigma = \frac{M}{Z} \qquad (3)$$

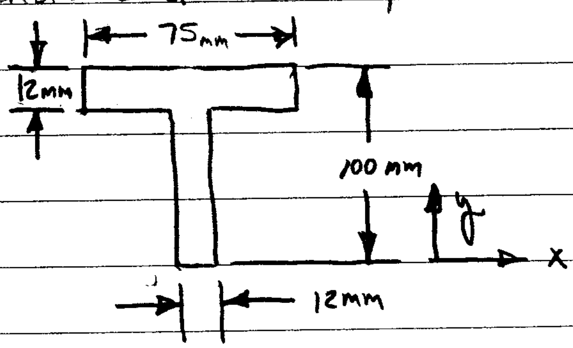
Where

$Z \triangleq$ section modulus

$$Z = \frac{I}{c} \qquad (4)$$

## Example

The cross-section below is subject to a bending moment of 1600 N.m which produces tension in the top fibers. Find the maximum tensile and compressive bending stresses.




We must first determine the location of the neutral axis,

$$\bar{y} = \frac{\Sigma y_i A_i}{A_{total}} \qquad (5)$$

$$\bar{y} = \frac{(12\,mm)(75\,mm)(94\,mm) + (88\,mm)(12\,mm)(44\,mm)}{(12\,mm)(75\,mm) + (88\,mm)(12\,mm)}$$

$$\bar{y} = 67.0\ mm$$

The neutral axis is located 67.0 mm above the bottom of the cross-section. I for the cross-section is calculated as,

$$I_x = \sum I_i + A_i \bar{y}_i^{\,2} \qquad \text{(Parallel-Axis Theorem)} \quad (6)$$

The second moment of a rectangular cross-section is,

$$I = \frac{bh^3}{12} \qquad (7)$$

where

$b \hat{=}$ section width
$h \hat{=}$ section height

The second moment of the cross-section in question is,

$$I_x = \overset{10800}{\frac{(75\,mm)(12\,mm)^3}{12}} + \overset{656\,100}{(12\,mm)(75\,mm)(27\,mm)^2} + \overset{681472}{\frac{(12\,mm)(88\,mm)^3}{12}} + (88\,mm)(12\,mm)(23\,mm)^2$$

$$I_x \doteq 1.91 \times 10^6\ mm^4$$

Bending stress is then calculated as,

$$\sigma = \frac{Mc}{I}$$

$$\sigma = \frac{(1600\ N.m)(33\ mm)}{(1.91 \times 10^6\ mm^4)} \left(\frac{1000\ mm}{1\ m}\right)^3 \left(\frac{1\ MPa \cdot m^2}{1{,}000{,}000\ N}\right)$$

$$\sigma = 27.6\ MPa \quad (\text{tension})$$

$$\sigma = \frac{(-1600\ N.m)(67\ mm)}{(1.91 \times 10^6\ mm^4)} \left(\frac{1000\ mm}{1\ m}\right)^3 \left(\frac{1\ MPa \cdot m^2}{1{,}000{,}000\ N}\right)$$

$$\sigma = -56.0\ MPa \quad (\text{compression})$$

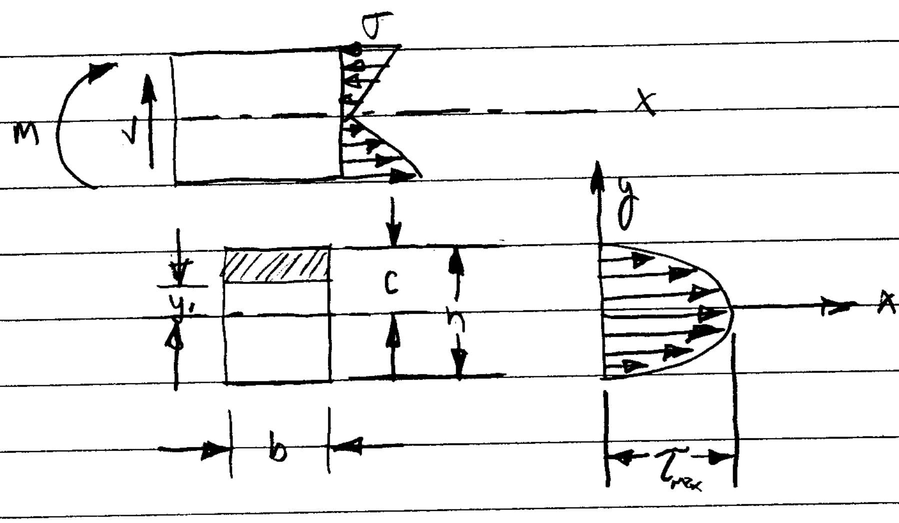
## Shear Stress in Beams

Most beams have both shear forces and bending moments present. We can utilize the same normal bending stress distribution as before. The transverse shear stress can be calculated as,

$$\tau = \frac{VQ}{Ib} \tag{8}$$

where

$\tau \triangleq$ transverse shear stress
$V \triangleq$ shear force
$Q \triangleq$ first moment of the area above the point in question
$b \triangleq$ section width at point in question




Recall that the first moment of the area above the point in question is,

$$Q = \int_{y_1}^{c} y\, dA \qquad (9)$$

and for a rectangular cross-section,

$$Q = \frac{b}{2}\left(c^2 - y_1^2\right) \qquad (10)$$

Substituting eq. 10 into eq. 8 we have,

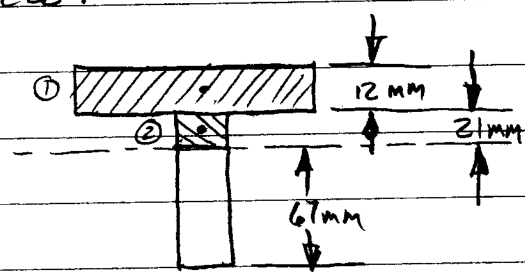
$$\tau = \frac{V}{2I}\left(c^2 - y_1^2\right) \qquad (11)$$

And from this we find the maximum shear stress occurs when $y_1 = 0$, or

$$\tau_{max} = \frac{3V}{2A} \quad \text{(for rectangular cross-sections)} \qquad (12)$$

## Example

Assume the cross-section from the previous example is subjected to a 20,000 N shear force in the −y direction. What is the maximum transverse shearing stress?




$$Q = \int_{y_1}^{c} y\,dA$$

$$Q = \Sigma \bar{y}_i A_i$$

$$Q = (12\,mm)(75\,mm)(27\,mm) + (12\,mm)(21\,mm)(10.5\,mm)$$

$$Q = 26950\ mm^3$$

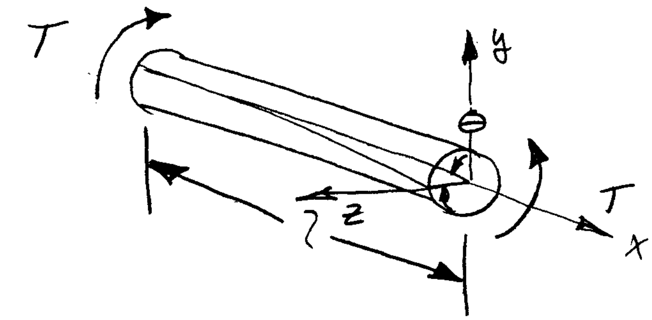
the maximum transverse shear stress is calculated as,

$$\tau = \frac{VQ}{Ib}$$

$$\tau_{max} = \frac{(20{,}000\,N)(26{,}950\,mm^3)}{(1.91\times10^6\,mm^4)(12\,mm)}\left(\frac{1000\,mm}{1\,m}\right)^2\left(\frac{1\,MPa\ m^2}{1{,}000{,}000\,N}\right)$$

$$\tau_{max} = 23.5\ MPa$$

## Torsional Shear Stress




The angle of twist for a solid round bar is,

$$\theta = \frac{Tl}{GJ}$$

where

$\theta \triangleq$ angle of twist
$T \triangleq$ applied torque
$l \triangleq$ length of round
$G \triangleq$ shear modulus
$J \triangleq$ polar second moment of area

Shear stress in the round bar is given by

$$\tau = \frac{T\rho}{J}$$

where

$\tau \triangleq$ torsional shear stress
$\rho \triangleq$ radius from centroid of cross-section

For solid round cross-sections, the polar second moment of the area is given as,

$$J = \frac{\pi d_o^4}{32}$$

where

$d_o \triangleq$ outside bar diameter

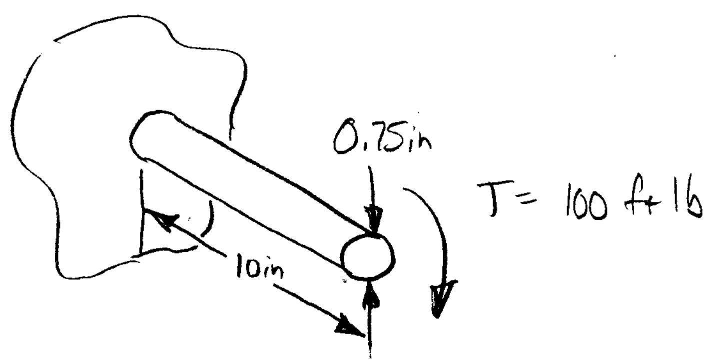
for hollow round sections,

$$J = \frac{\pi}{32}\left(d_o^4 - d_i^4\right)$$

where

$d_i \triangleq$ inside tube diameter

Example




A mild steel rod (0.75 in. diameter) is subjected to a torque of 100 ftlb. If the length is 10.0 in, what is the angular deflection at the free end, and the maximum shear stress in the rod?

Polar second moment,

$$J = \frac{\pi}{32}(0.75\,in)^4$$

$$J = 0.0311\ in^4$$

Maximum shear stress,

$$\tau = \frac{T\rho}{J}$$

$$\tau = \frac{(100\ ft\,lb)(0.375\,in)}{(0.0311\,in^4)}\left(\frac{12\,in}{1\,ft}\right)$$

$$\tau = 14{,}500\ \frac{lb}{in^2}$$

Angle of twist,

$$\theta = \frac{Tl}{GJ}$$

$$G = \frac{E}{2(1+\nu)}$$

$$G = \frac{(30.0\times10^6\ \frac{lb}{in^2})}{2(1+0.3)}$$

$$G = 11.5\times10^6\ \frac{lb}{in^2}$$

$$\theta = \frac{(100\ ft\,lb)(10.0\,in)}{(11.5\times10^6\ \frac{lb}{in^2})(0.0311\,in^4)}\left(\frac{12\,in}{1\,ft}\right)$$

$$\theta = 0.0336\ \text{radians}\ (1.93^\circ)$$