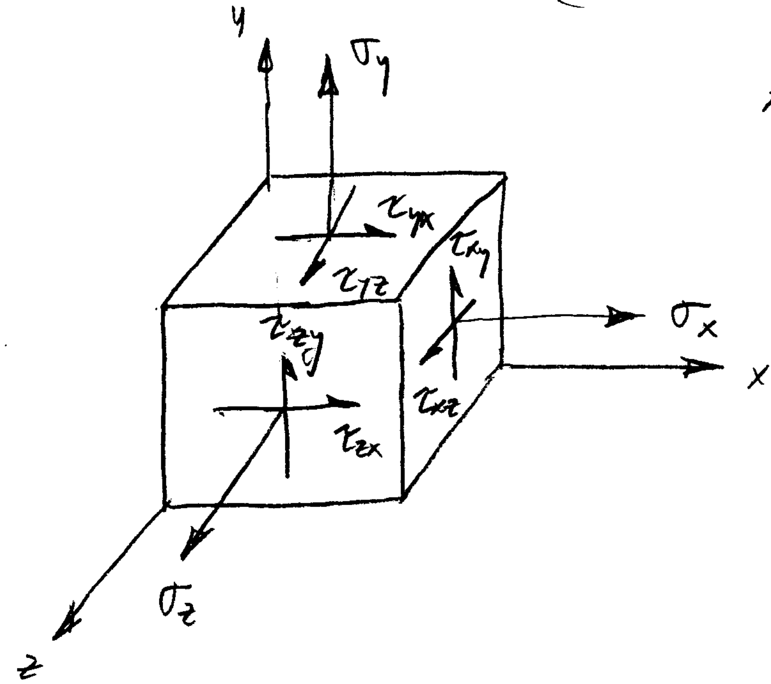
Lecture 4

Stress Components (4-4 to 4-8)

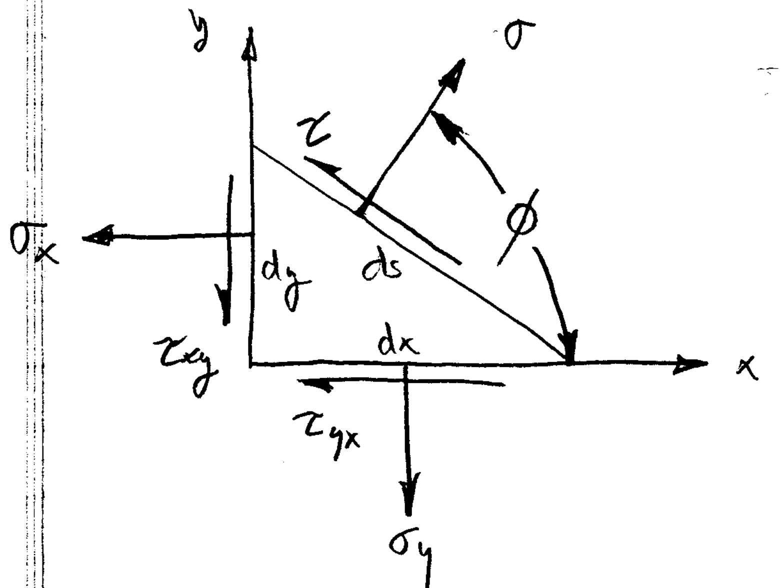


* Note correction of Fig 2-1 in text τ_{zy}, not τ_{xy} on face in z-direction

$\sigma_x, \sigma_y, \sigma_z$ – normal stresses

$\tau_{xy}, \tau_{yx}, \tau_{xz}, \tau_{zx}, \tau_{yz}, \tau_{zy}$ – shear stresses

Mohr's Circle for Biaxial Stress (Plane Stress)



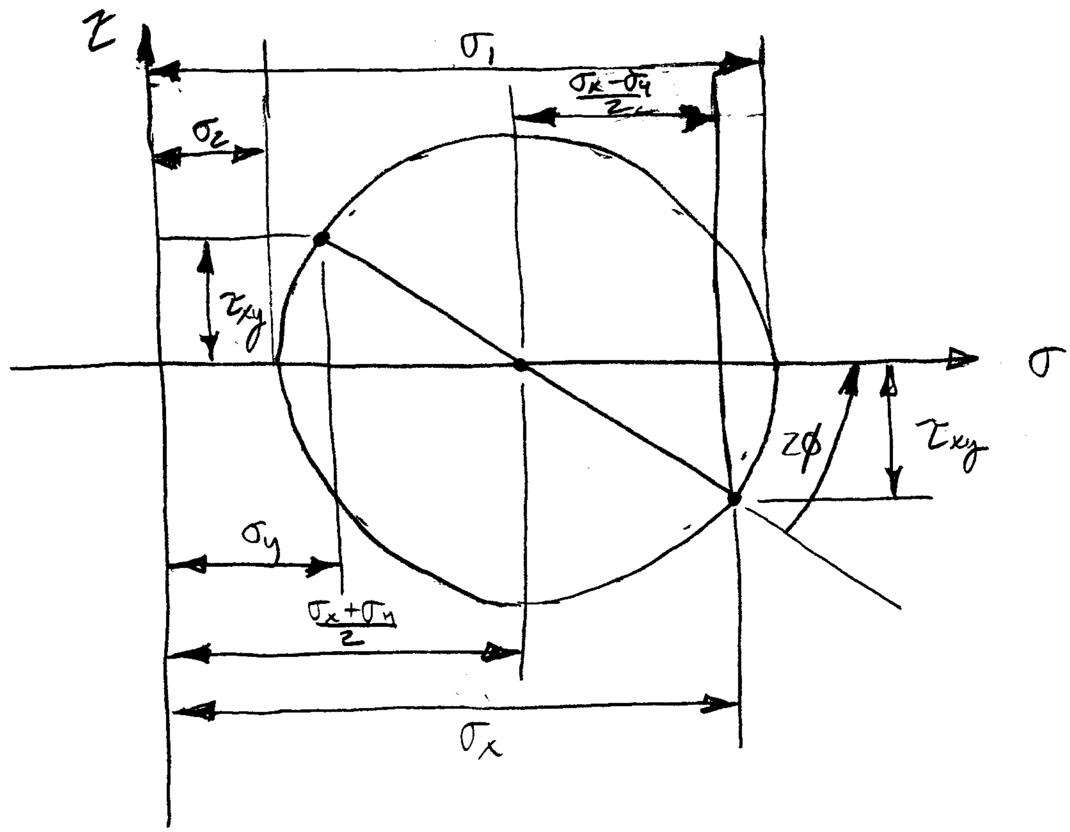
When we consider stresses in 2-D the element can be oriented such that the shear stress is zero (maximum principal stress) or a maximum (maximum shear stress).

Stress at any Orientation

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy}\sin 2\phi \quad (1)$$

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy}\cos 2\phi \quad (2)$$

Mohr's Circle Diagram



Maximum Principal Stress

Differentiating the normal stress equation w.r.t. ϕ and setting the resultant equal to zero,

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad (3)$$

Substituting the angle of 2ϕ from eq. 3 into eq. 1, an equation for describing the maximum principal stress is obtained.

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (4)$$

In a similar manner we can differentiate eq. 2, set it equal to zero, and obtain an expression which defines an angle ϕ that describes the element orientation of maximum sheare stress,

$$\tan 2\phi = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \qquad (5)$$

Combining eqs. 2 and 5 we obain an expession for maximum shear stress,

$$\tau_1, \tau_2 = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (6)$$

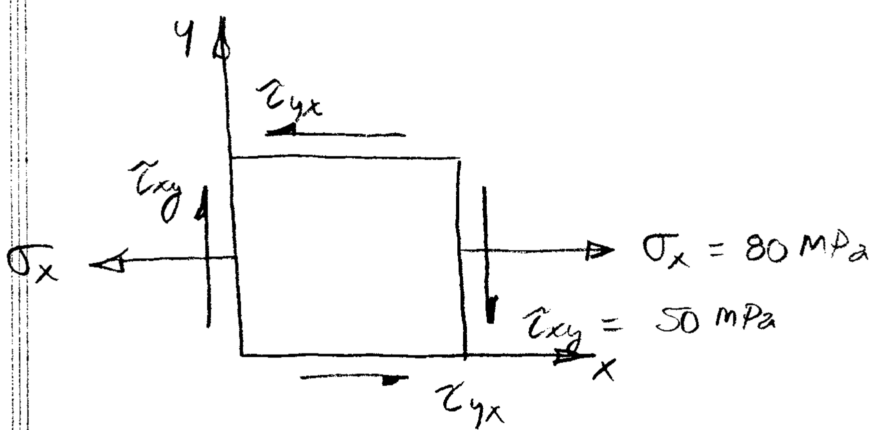
<u>Triaxial Stress</u>

When we consider stresses in 3-D the elemental volume can be oriented in space such that the shear stresses on all faces are zero (principal normal stress)

Example:

A stress element has $\sigma_x = 80$ MPa and $\tau_{xy} = 50$ MPa cw. Find the principal stresses and directions and show these on a stress element correctly aligned with respect to the xy system. Draw another stress element to show τ_{max}, find the corresponding normal stresses and label the drawing

Initial Stress State



$$\sigma_1, \sigma_2 = \frac{80\text{ MPa}}{2} \pm \sqrt{\left(\frac{80\text{ MPa}}{2}\right)^2 + (50\text{ MPa})^2}$$

$$\sigma_1 = 104.0\text{ MPa}$$

$$\sigma_2 = -24.0\text{ MPa}$$

$$\tau_{max} = 64.0\text{ MPa}$$

$$\tan 2\phi_\sigma = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

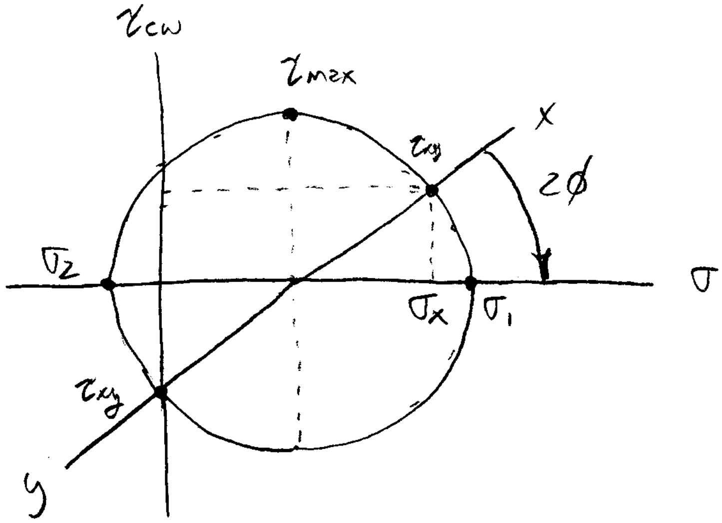
$$\phi_\sigma = \frac{\tan^{-1} \frac{2(50\ MPa)}{80\ MPa}}{2}$$

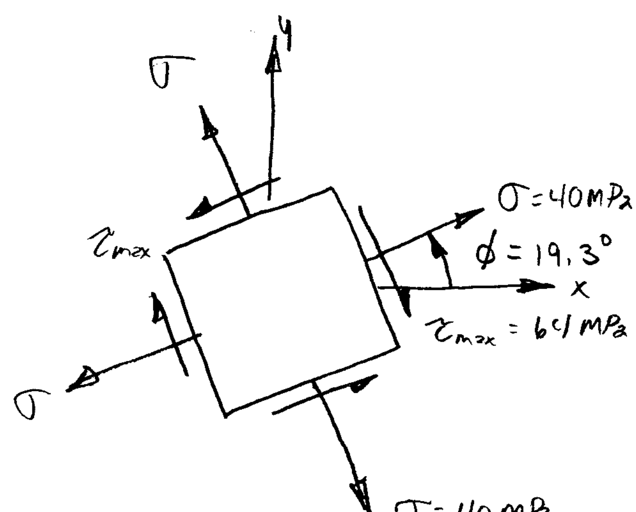
$$\phi_\sigma = 25.7^\circ \quad (\text{principal stress})$$

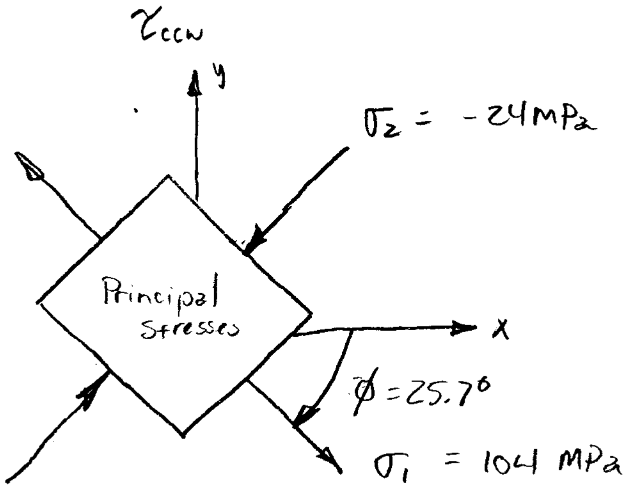
$$\tan 2\phi_\tau = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\phi_\tau = \frac{\tan^{-1} \frac{-80\ MPa}{2(50\ MPa)}}{2}$$

$$\phi_\tau = -19.3^\circ \quad (\text{max. shear stress})$$



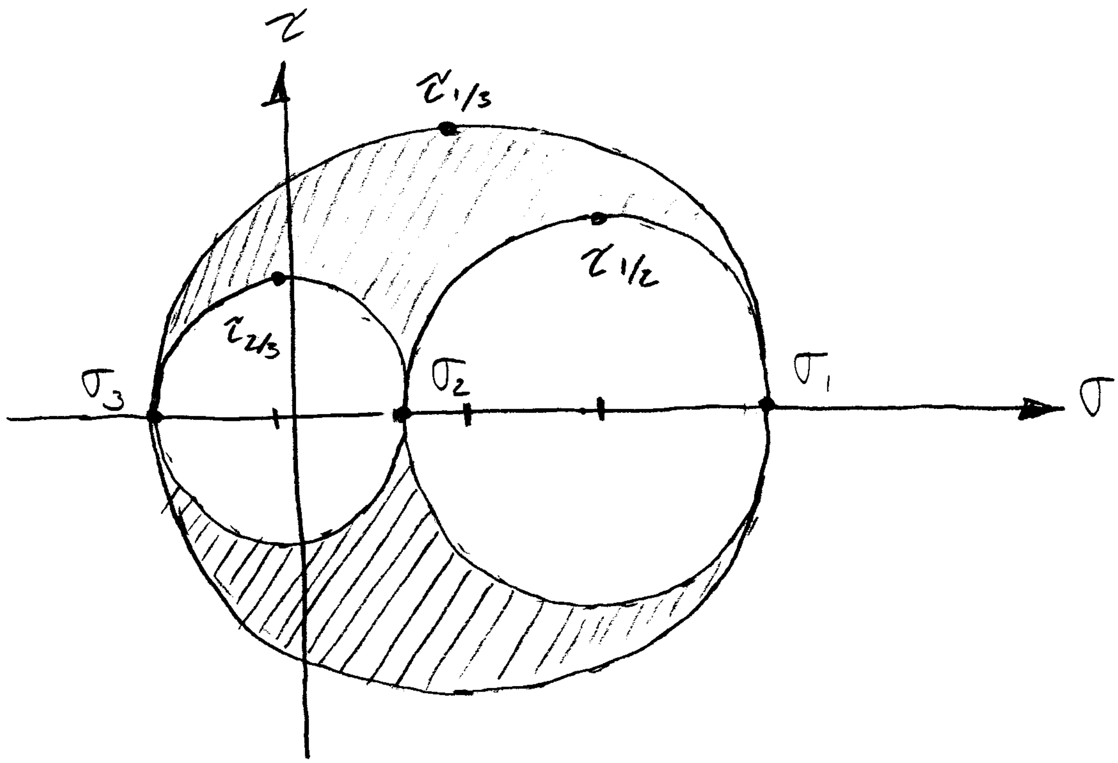




To determine the principal stresses we must solve for the roots of the stress cubic equation,

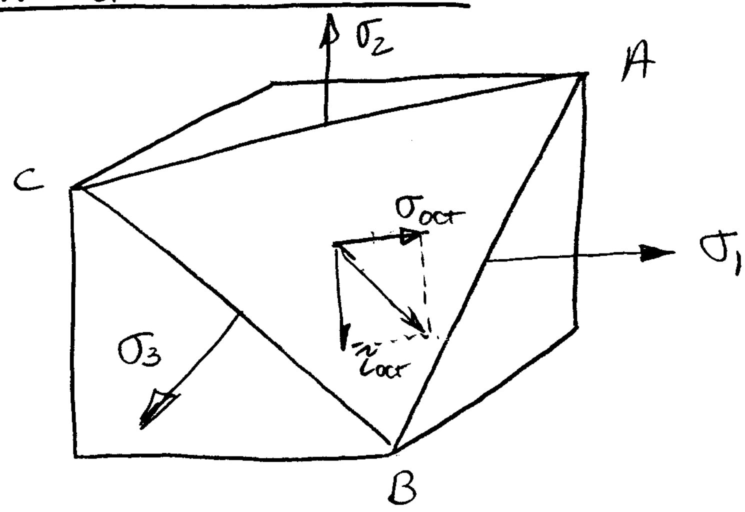
$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma$$
$$- (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2) = 0 \quad (7)$$

Mohr's Circle in 3-D



$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}, \quad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \quad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} \quad (8)$$

Octahedral Stresses



By passing planes through the vertices of the pricipal stress element a resultant force can be found which places the element in equilibrium. This plane is called the octahedral plane, there are eight of these. the octahedral normal and shear stresses are calculated as,

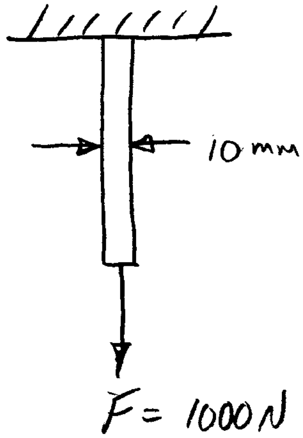
$$\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) \quad (9)$$

$$\tau_{oct} = \frac{2}{3}(\tau_{1/2}^2 + \tau_{2/3}^2 + \tau_{1/3}^2)^{\frac{1}{2}}$$

$$= \frac{1}{3}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)\right]^{\frac{1}{2}} \quad (10)$$

Uniformly Distributed Stresses

Axial Stress (pure tension or compression)

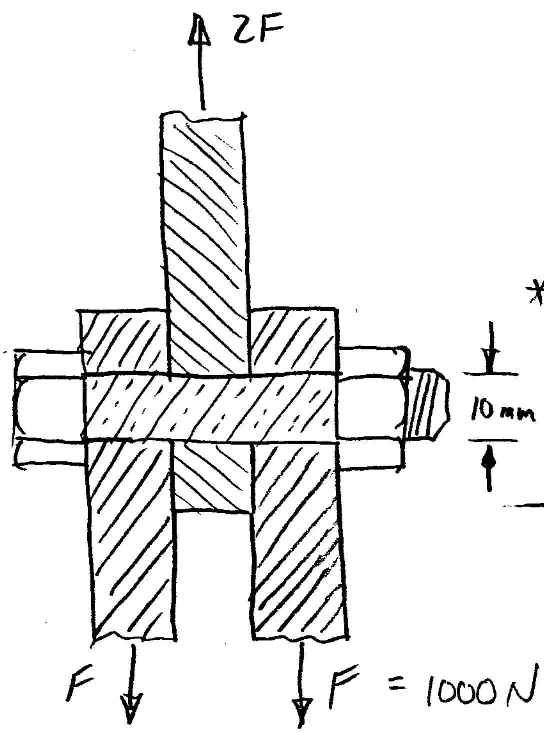


$$\sigma = \frac{F}{A} \tag{11}$$

$$\underline{12700}\ kPa = \frac{(1000\,N)}{\frac{\pi}{4}(0.01\,m)^2}\left(\frac{1\,kPa \cdot m^2}{1000\,N}\right)$$

* Bar is straight and of homogeneous material.
* Line of action of force contain centroid of cross-section.
* Section taken remote from end and away from discontinuity or abrudt changes in cross-section

Normal Shear Stress



$$\tau = \frac{F}{A} \tag{12}$$

* Good assumption for bolt or pin in double shear.

$$\_\_\_\_\_\ kPa = \frac{(1000\,N)}{\frac{\pi}{4}(0.01\,m)^2}\left(\frac{1\,kPa \cdot m^2}{1000\,N}\right)$$

Elastic Strain

Strain - the amount of elongation or stretch

Unit strain - elongation per unit length

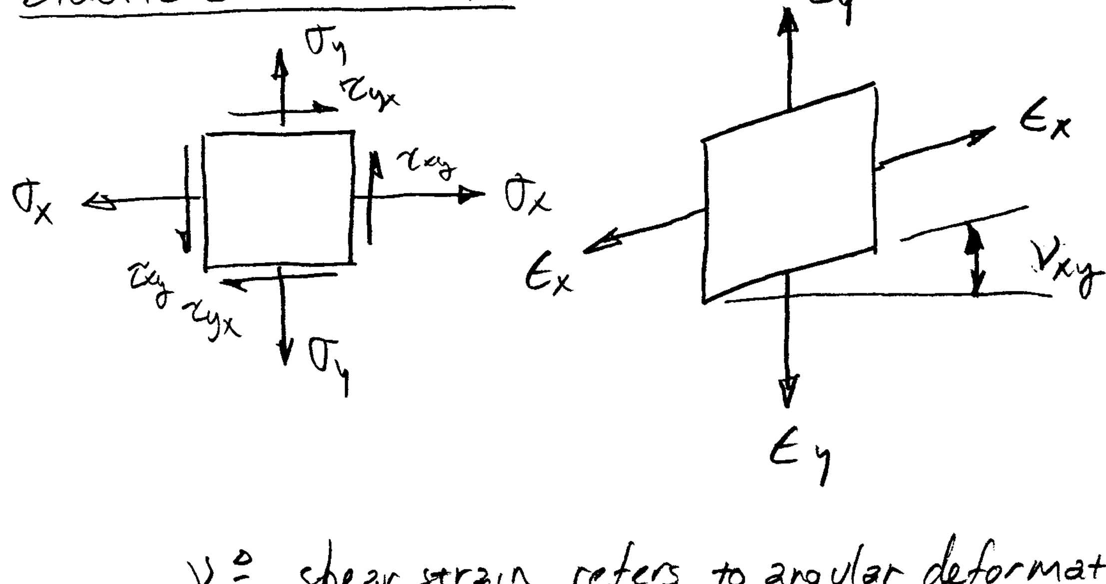
* Convention dictates that "unit strain" is usually referred to as "strain."

$$\epsilon = \frac{\delta}{l} \qquad (13)$$

where

$\epsilon \triangleq$ unit strain
$\delta \triangleq$ deflection
$l \triangleq$ initial length

Elastic Shear Strain



$\gamma \triangleq$ shear strain, refers to angular deformation

Modulus of Elasticity

Hooke's Law - stress is proportional to strain, material that obey this law are elastic

The constant of proportionality between normal stress and normal strain is termed the "Modulus of Elasticity" or "Young's Modulus."

$$E = \frac{\sigma}{\epsilon} \quad \text{or} \quad \sigma = E\epsilon \qquad (14)$$

where

$E \triangleq$ modulus of elasticity, a material property

The constant of proportionality between shear stress and shear strain is termed the "Shear Modulus of Elasticity" or "Modulus of Rigidity."

$$G = \frac{\tau}{\gamma} \quad \text{or} \quad \tau = G\gamma \qquad (15)$$

where

$G \triangleq$ shear modulus of elasticity, a material property

By substituting eq. 13 into eq. 14,

$$\delta = \frac{Fl}{AE} \tag{16}$$

which describes the elongation of a bar loaded in axial tension or compression.

The ratio of lateral to axial strains is termed Poisson's Ratio,

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

The elastic constants are related to each other as,

$$E = 2G(1+\nu)$$

Elastic Stress-Strain Relationships

	Principal Strains	Principal Stresses
Uniaxial	$\epsilon_1 = \frac{\sigma_1}{E}$	$\sigma_1 = E\epsilon_1$
	$\epsilon_2 = -\nu\epsilon_1$	$\sigma_2 = 0$
	$\epsilon_3 = -\nu\epsilon_1$	$\sigma_3 = 0$

	Principal Strains	Principal Stresses
Biaxial	$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E}$	$\sigma_1 = \frac{E(\epsilon_1 + \nu\epsilon_2)}{1-\nu^2}$
	$\epsilon_2 = \frac{\sigma_2}{E} - \frac{\nu\sigma_1}{E}$	$\sigma_2 = \frac{E(\epsilon_2 + \nu\epsilon_1)}{1-\nu^2}$
	$\epsilon_3 = -\frac{\nu\sigma_1}{E} - \frac{\nu\sigma_2}{E}$	$\sigma_3 = 0$
Triaxial	$\epsilon_1 = \frac{\sigma_1}{E} - \frac{\nu\sigma_2}{E} - \frac{\nu\sigma_3}{E}$	$\sigma_1 = \frac{E\epsilon_1(1-\nu) + \nu E(\epsilon_2+\epsilon_3)}{1-\nu-2\nu^2}$
	$\epsilon_2 = \frac{\sigma_2}{E} - \frac{\nu\sigma_1}{E} - \frac{\nu\sigma_3}{E}$	$\sigma_2 = \frac{E\epsilon_2(1-\nu) + \nu E(\epsilon_1+\epsilon_3)}{1-\nu-2\nu^2}$
	$\epsilon_3 = \frac{\sigma_3}{E} - \frac{\nu\sigma_1}{E} - \frac{\nu\sigma_2}{E}$	$\sigma_3 = \frac{E\epsilon_3(1-\nu) + \nu E(\epsilon_1+\epsilon_2)}{1-\nu-2\nu^2}$

Equilibrium

Objects are said to be in static equilibrium when

$$\Sigma F = 0$$

$$\Sigma M = 0$$

Free Body Diagrams

1. The diagram establishes the direction of reference axes, magnitudes and direction of known forces, and helps when assuming direction of unknowns.

2. The diagram provides a method of communicating your thought clearly.

3. The diagram aids in clarifying the problem.

4. The diagram helps in setting up the mathematical relationships to solve for unknowns.