Lecture 10

Endurance-Limit Modifying Factors (7-6)

$$S_e = k_a k_b k_c k_d k_e S_e' \qquad (1)$$

where,

$S_e' \triangleq$ endurance limit of test speciment
$S_e \triangleq$ endurence limit of mechanical element
$k_a \triangleq$ surface factor
$k_b \triangleq$ size factor
$k_c \triangleq$ load factor
$k_d \triangleq$ temperature factor
$k_e \triangleq$ miscellaneous-effects factor

Surface Factor (k_a)

$$k_a = a S_{ut}^{b}$$

Surface Finish	Factor a (kpsi)	Factor a (MPa)	Exponent b
ground	1.34	1.58	−0.085
machined or cold-drawn	2.70	4.51	−0.265
hot-rolled	14.4	57.7	−0.718
as forged	39.9	272.0	−0.995

Size Factor (k_b)

For round bars in bending and torsion.

$$k_b = \begin{cases} \left(\dfrac{d}{0.3}\right)^{-0.1133} & \text{in} \quad 0.11 \le d \le 2.0 \text{ in} \\ \left(\dfrac{d}{7.62}\right)^{-0.1133} & \text{mm} \quad 2.79 \le d \le 51 \text{ mm} \\ 0.60 \text{ to } 0.75 & d > 2.0 \text{ in } (51 \text{ mm}) \end{cases} \qquad (2)$$

$k_b = 1$ for axial loading (No size effect?)

For other cross-sections we employ an "effective dimension", d_e. The method used in this case equates the volume of stressed material to the same volume in a rotating beam specimen at and above 95% of the maximum stress. The 95% stress area is,

$$A_{0.95\sigma} = \frac{\pi}{4}\left[d^2 - (0.95d)^2\right] = 0.0766 d^2 \qquad (3)$$

This approach is valid for rotating round bars and hollow rounds.

For non rotating solid or hollow rounds the stress area is considered to be twice the area outside of parallel cords placed at a distance of 0.95D where D is the diameter,

$$A_{0.95\sigma} = 0.0105\,D^2 \qquad (4)$$

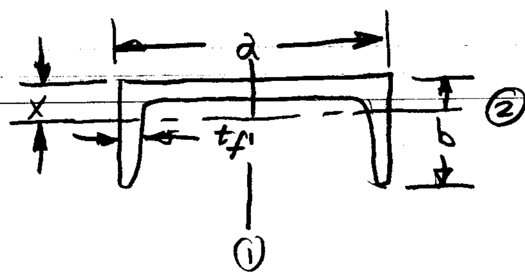
Setting eqs. 3 and 4 equal,

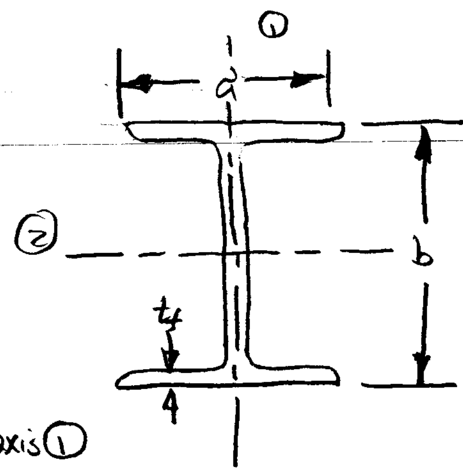
$$d_e = 0.370\,D \qquad (5)$$

For a rectangular cross-section,

$$d_e = 0.808\,(bh)^{\frac{1}{2}} \qquad (6)$$

For other shapes such as channels and I-beams,





Wide Flange

$$A_{0.95\sigma} = \begin{cases} 0.05\,ab & \text{axis ①} \\ 0.052\,xa + 0.1\,t_f\,(b-x) & \text{axis ②} \end{cases}$$

I-beam

$$A_{0.95\sigma} = \begin{cases} 0.10\,a\,t_f & \text{axis ①} \\ 0.05\,ba \quad t_f > 0.025a & \text{axis ②} \end{cases}$$

Load Factors (k_c)

$$k_c = \begin{cases} 0.923 & \text{axial loading} \quad S_{ut} \le 220\,\text{kpsi}\ (1520\,\text{MPa}) \\ 1.0 & \text{axial loading} \quad S_{ut} > 220\,\text{kpsi}\ (1520\,\text{MPa}) \\ 1.0 & \text{bending} \\ 0.577 & \text{torsion and shear} \end{cases}$$

Temperature Factors (k_d)

If the rotating beam endurance limit is known at room temperature use,

$$k_d = \frac{S_T}{S_{RT}} \qquad (7)$$

from Table 7-5 of text. If not, then compute it using

$$S_e' = \begin{cases} 0.504\,S_{ut} & S_{ut} \le 200\,\text{kpsi}\ (1400\,\text{MPa}) \\ 100\,\text{kpsi} & S_{ut} > 200\,\text{kpsi} \\ 700\,\text{MPa} & S_{ut} > 1400\,\text{MPa} \end{cases}$$

and calculate the temperature corrected stress from Table 7-5. $k_d = 1$ for the latter case,

Miscellaneous - Effects Factors (k_e)

Stress Concentration Factors

For ductile materials at less than 10^3 cycles the load is considered static and therefore stress concentration factors need not be employed. K_f should be used for design lifes greater then 10^6 cycles. Two approaches exist for design lives between 10^3 and 10^6 cycles. The first,

$$k_e = \frac{1}{K_f} \tag{8}$$

the second is to reduce K_f for lives less than 10^6 cycles,

$$K_f' = a N^b \tag{9}$$

where

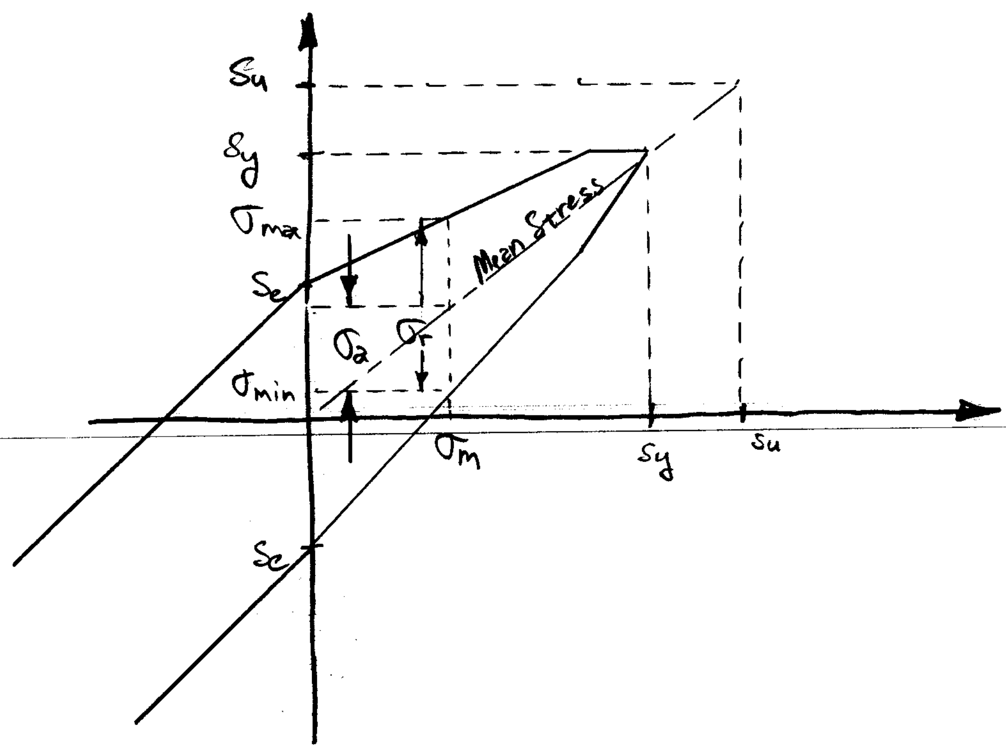
$$a = \frac{1}{K_f} \tag{10}$$

$$b = -\frac{1}{3} \log \frac{1}{K_f} \tag{11}$$

$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad (12)$$

$$A = \frac{\sigma_a}{\sigma_m} \qquad (13)$$

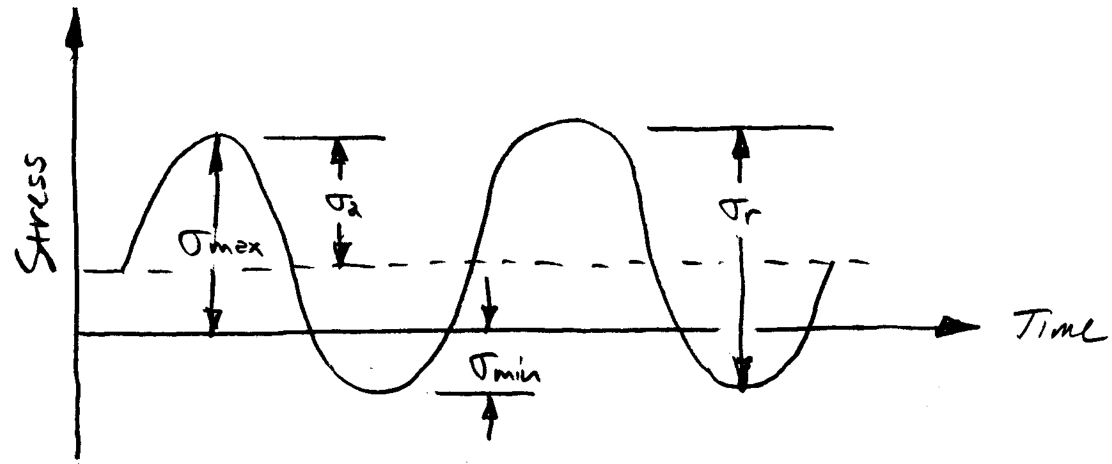
Now that we have characterized this non-zero mean stress, how do we handle it in design? One test in general use is the Modified Goodman relationship,



There are three additional relationships that are used with some degree of frequency,

1) Yield Method
2) Gerber Method
3) Soderberg Method

Fluctuating Stress (Sec. 7-12 through 7-16)



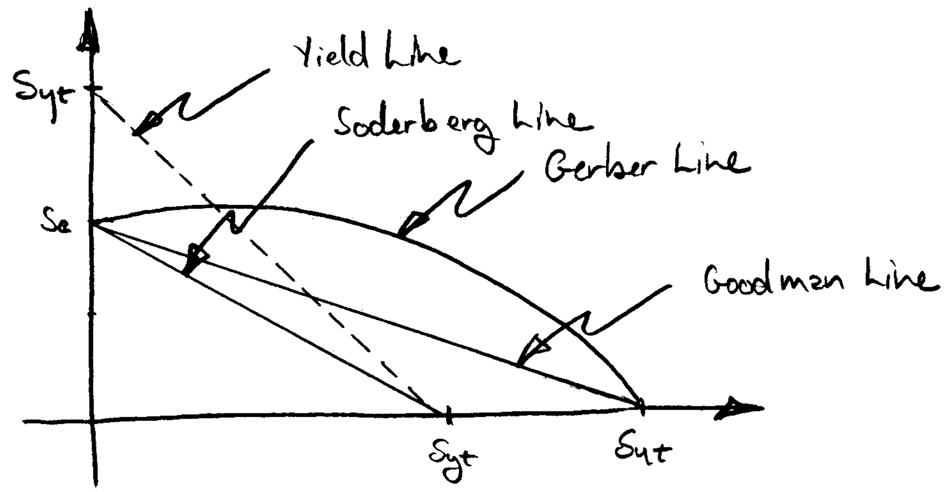
$\sigma_{min} \triangleq$ minimum stress
$\sigma_{max} \triangleq$ maximum stress
$\sigma_a \triangleq$ stress amplitude
$\sigma_r \triangleq$ Stress range
$\sigma_m \triangleq$ mean stress

The following relationships should be evident

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \tag{14}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \tag{15}$$

Two additional variables are introduced to describe the nature of fluctuating stress, The figure belows shows a comparison of the three methods,



The relationships used for design are as follows,

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1 \quad \text{(Soderberg)} \qquad (16)$$

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \quad \text{(Goodman)} \qquad (17)$$

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1 \quad \text{(Gerber)} \qquad (18)$$

$$\frac{S_a}{S_{yt}} + \frac{S_m}{S_{yt}} = 1 \quad \text{(Yielding)} \qquad (19)$$

Replacing the stresses S_a and S_m with σ_a and σ_m,

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{n} \quad \text{(Soderberg)}$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad \text{(Goodmen)}$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \quad \text{(Gerber)}$$

where n is the factor of safety.

Example

A leaf spring is used to maintain contact between a roller-follower and a cam, the range of motion of the follower is fixed, however the preload on the spring can be increased to prevent floater jump at higher speeds.

The spring is a steel cantilever 32.0 in. long, 2.0 in. wide and 0.25 in thick. The strengths are $S_u = 150$ kpsi, $S_y = 127$ kpsi and $S_e = 28$ kpsi. The total cam motion is 2.0 in and we wish to load the spring 5.0 in for high speed operation. Determine the factor of safety using the Goodman criteria.

Solution

The second moment of the area is,

$$I = \frac{bh^3}{12} = \frac{(2.0\,in)(0.25\,in)^3}{12} = 0.00260\,in^4$$

and the deflection is related to the force as,

$$k = \frac{F}{y} = \frac{3EI}{l^3} = \frac{3(30\times10^6 psi)(0.00260\,in^4)}{(32.0\,in)^3}$$

$$k = 7.14\,\frac{lb}{in}$$

The bending stress caused by a deflection of δ in. is,

$$\sigma = \frac{Mc}{I} = \frac{(7.14\,\frac{lb}{in})(32.0\,in)(0.125\,in)\,\delta}{(0.00260\,in^4)}$$

$$\sigma = (11{,}000\,\tfrac{lb}{in^3})\,\delta$$

Using the Goodman relationship,

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{n}$$

$$\frac{(11{,}000 \frac{lb}{in^3})(1.0\,in)}{(28{,}000 \frac{lb}{in^2})} + \frac{(11{,}000 \frac{lb}{in^3})(5.0\,in)}{(150{,}000 \frac{lb}{in^2})} = \frac{1}{n}$$

$$\Rightarrow \quad n = 1.32$$

* This is the governing "design factor" for the spring operated under high speeds conditions.

Torsional Fatigue Strength for Pulsating Stresses

In construction diagrams,

$$S_{su} = 0.67\, S_{ut}$$

Combinations of Loadings (Sec. 7-15)

What happens when axial, bending and torsional loads occur at the same time? The answer is summarized as follows:

1. Use the fully corrected endurance limit for bending (S_e).

2. Apply the appropriate stress concentration factors to the alternating components of torsional stress, bending stress and axial stress.

3. Multiply any alternating axial stress components by $\frac{1}{k_c} = 1.083$.

4. Find principal stresses from Mohr's Circle.

5. Find the von Mises alternating stress σ_a'.

6. Compare σ_a' to S_e to find the factor of safety.

<u>Cumulative Fatigue Damage</u> (Sec 7-16)

Suppose a part is subjected to a load of σ_1 for n_1 cycles, σ_2 for n_2 cycles and σ_3 for n_3 cycles. How do we estimate the life of the part under fluctuating load conditions? The Palmgren-Miner cycle-ratio summation theory may be used.

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \cdots + \frac{n_i}{N_i} = C$$

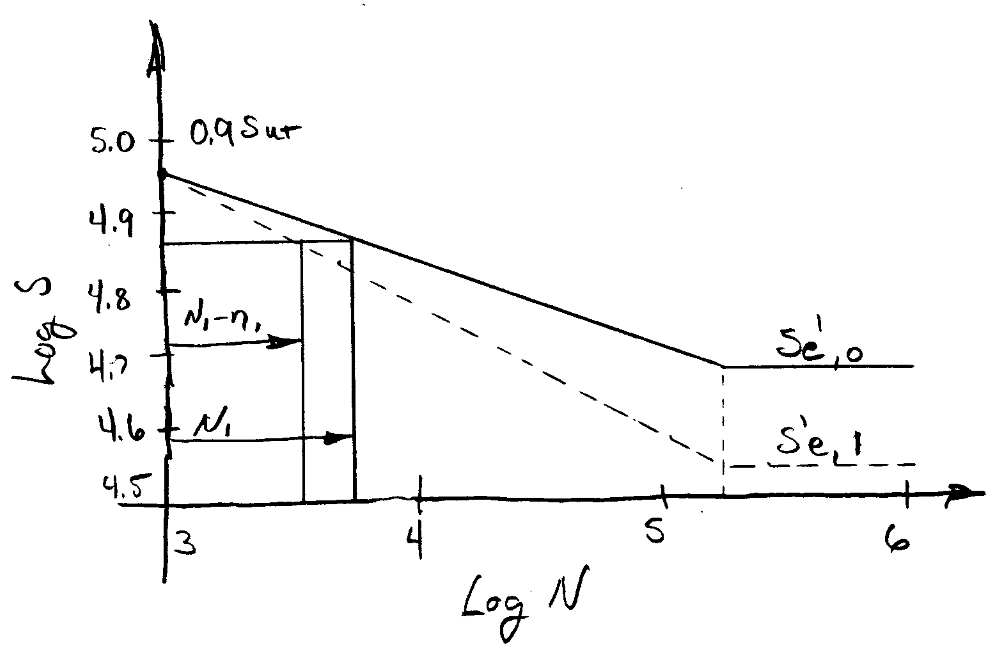
where,

$n_i \stackrel{\circ}{=}$ number of stress cycles at level i
$N_i \stackrel{\circ}{=}$ number of cycles to failure at level i
$C \stackrel{\circ}{=}$ constant

C is determined by experimental methods and is found to range from 0.7 to 2.2, Most authors recommend using $C = 1$.

* Miner's Rule does not account for the order in which loads are applied,

* Mason's method avoid this problem by making sure all $\log S - \log N$ line converge to $0.9\,S_{ut}$ at 10^3 cycles. These lines must be constructed in historical order,



* Mason's method is preferred to Miner's Rule because it accounts for the order of loading.